ADDITIONAL LOAD FROM RADIAL MISALIGNMENT OF SHAFTS AT THEIR COUPLING

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ABSTRACT

A methodology for determining of additional load of shafts from deformation due to radial misalignment at their coupling is proposed. Shafts connected with rigid compensating coupling with cross coulisse (Oldham coupling) and with sleeve coupling are considered. The shafts displacement due to the additional load is determined for two cases: when supports are considered as perfect solid and when supports are considered as flexible with a radial clearance in the bearing. The elastic deformation of coupling sleeve is determined.

Key words: misaligned shafts, coupling, rolling bearing, displacement, strain.

INTRODUCTION

To connect the shafts with a coupling they need to be closely aligned. This requirement is difficult to achieve due to deviations from the axial position caused by defective production of constructive elements of the couplings, and practically unavoidable displacements (offsets) due to the montage of machines and mechanisms. Also deviations from the deformations of the assembly shafts caused by operational loads, thermal effects, and more are obtained. The presence of misalignment causes additional load of connecting shafts, their supports and details of the coupling.

To determine the additional load of the shafts, of the supports and the parts of the couplings when connecting the radially offset shafts it is necessary to define the displacements from the elastic deformation of the shafts, the supports and the elements of the couplings.

1. COMPENSATION CAPACITY OF COUPLINGS AND ADDITIONAL LOAD OF DRIVING ELEMENTS

Coupling of radial misaligned shafts with two types of couplings – rigid compensating coupling with cross coulisse (Oldham coupling) and stationary sleeve coupling are considered. When connecting the radial displaced shafts an additional radial force F is generated and this force loads the shafts. This additional force depends on the type and size of the coupling, the sizes of the shafts, the size of the transmitted peripheral force and on size of the radial displacement r (Fig. 1). This force must be taken into account when verification calculations of the shafts are carried out because it can cause vibration and early failure of connecting components – usually bearings or shafts.

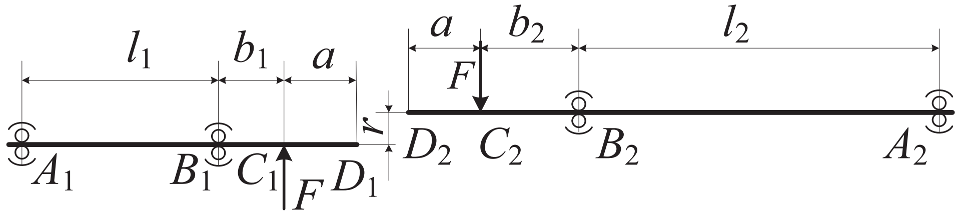

Figure 1: Situation, sizes and loading of radial misaligned shafts

When connecting the radial displaced shafts with *Oldham coupling* the additional force is small, since the construction of the coupling allows slip of the projections of the coulisse in diametrically channels of the coupling discs. In this case, the additional force F, loading console parts of shafts is equal to the friction force F_f, which is generated by the friction of the coulisse in the grooves of the coupling discs and peripheral force F_t transmitted by the coupling, i.e.

$$F = F_f , \qquad (1)$$

where F is the force, loading console parts of shafts, N;

F_f – friction force, N.

The friction force can be determined by the following formula:

$$F_f = \mu . F_t , \qquad (2)$$

where μ is friction factor (for semidried friction can be $\mu = 0{,}2$);

F_t – transmitted peripheral force, N.

Then the force loading console parts of the shafts is obtained:

$$F = 0.2\, F_t \qquad (3)$$

When coupling of radial displaced shafts with stationary *sleeve coupling* the additional radial force can be determined if radial displacement value r and stiffness ratio k_Σ of the driving elements are known:

$$r = k_\Sigma . F \qquad (4)$$

where r is the radial displacement of coupling shafts, m;

k_Σ – sum of stiffness ratio of driving elements, m.N^{-1} (equal of vertical displacement in m from force $F = 1$ N).

In this case the radial displacement of the shafts r can be compensated by the elastic deformation of the shafts in their console parts $f_{B\Sigma}$, by deformation of the supports $w_{o\Sigma}$ and by deformation of the coupling sleeve $z_{c\Sigma}$, i. e.

$$r = f_{B\Sigma} + w_{o\Sigma} + z_{c\Sigma} \qquad (5)$$

If we express the displacements $f_{B\Sigma}$, $w_{o\Sigma}$ and $z_{c\Sigma}$ as a function of the additional radial force F and stiffness ratio $k_{B\Sigma}$, $k_{o\Sigma}$ и $k_{c\Sigma}$, respectively and the following relationship will be obtained:

$$z_{c\Sigma} = k_{B\Sigma} . F + k_{o\Sigma} . F + k_{c\Sigma} . F \qquad (6)$$

In this case the additional force is received:

$$F = \frac{r}{k_{B\Sigma} + k_{o\Sigma} + k_{c\Sigma}} . \qquad (7)$$

2. DISPLACEMETS DUE TO THE ELASTIC DEFORMATIONS OF DRIVING ELEMENTS GENERATED FROM THE ADDITIONAL LOAD FOR SLEEVE COUPLIG

2.1. Determination of the elastic deformations of the shafts

The elastic deformation of shafts $f_{B\Sigma}$ in the console parts (p. D_1 and p. D_2 – Fig.2) due to radial force F is a sum of displacements of the first shaft f_1 and of second shaft f_2:

$$f_{B\Sigma} = f_1 + f_2 . \qquad (8)$$

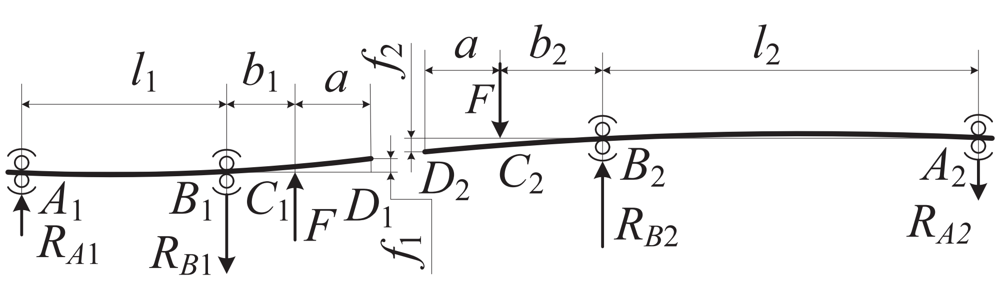

Figure 2: Displacement due to elastic deformations of the shafts loaded with additional radial force *F* in their console parts

Fig. 3 illustrates the displacement f_i (i = 1, 2) of the console part of a shaft and the type of elastic line of the shaft after its bending when supports are perfectly rigid. In these cases, usually rolling bearings are considered as perfectly rigid supports.

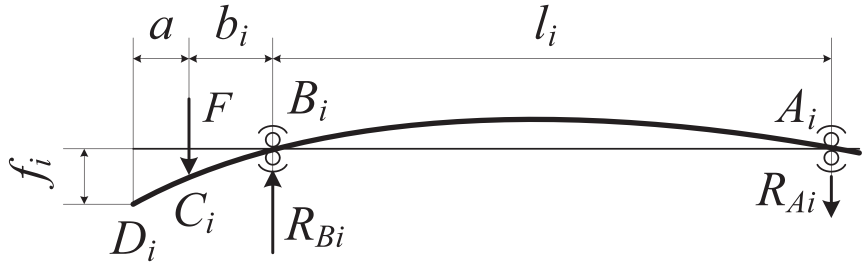

Figure 3: Displacement of shafts loaded with a force outside of supports when they are perfectly rigid

The displacement f_i can be determined according the following dependence:

$$f_i = \frac{F.(a+b_i).b_i.(l_i+b_i)}{3E.I_i}, \qquad \textbf{(9)}$$

where f_i is the shaft displacement in console part due to the additional force F, m;

F – additional force loaded the shaft, N;

E – modulus of elastic deformation for the shaft material, Pa;

I_i – moment of inertia of shaft section (i = 1, 2), m^4.

The moment of inertia of shaft section is determined by the known formula:

$$I_i = \frac{\pi.d_i^4}{64}, \qquad \textbf{(10)}$$

where d_i is diameter of the shaft (i = 1, 2), m;

a, b_i, l_i – distance according to Fig.2 and Fig.3 (i = 1, 2), m.

For determination of dependence (9) the formula for displacement in point C_i (f_c) (Christov et al. 1980) and the equality of relations of displacements in points C_i and D_i and corresponding lengths (b_i - a) and b_i (Fig. 3) were used, i.e.

$$f_c = \frac{F.b_i.(l_i+b_i)}{3.E.I_i} \qquad \textbf{(11)}$$

$$\frac{f_i}{f_c} = \frac{a+b_i}{b_i} \qquad \textbf{(12)}$$

Then, stiffness ratio $k_{\text{B}\Sigma}$ is received:

$$k_{\text{B}\Sigma} = \frac{f_{\text{B}\Sigma}}{F} \qquad \textbf{(13)}$$

2.2. Determination of shafts displacements due to supports deformation

Shafts displacement due to supports deformation $w_{o\Sigma}$ is a sum of displacement of the first shaft w_1 and displacement of the second shaft w_2 (Fig. 4), i.e.,

$$w_{o\Sigma} = w_1 + w_2. \qquad \textbf{(14)}$$

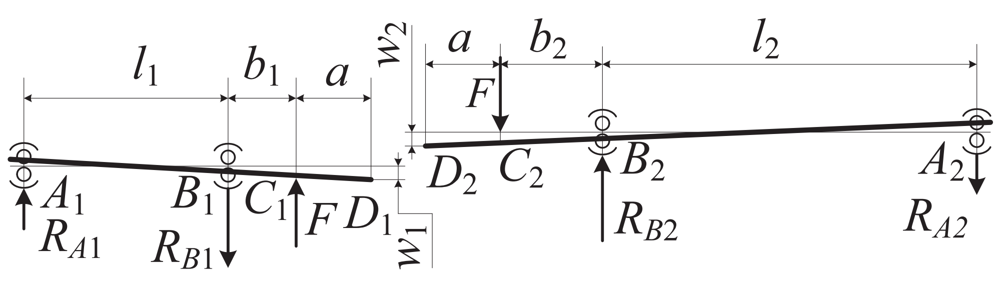

Figure 4: Displacement in console part of misaligned shafts due to supports deformation

The displacement w_i of console part of a perfectly rigid shaft (i = 1, 2) depending on support displacements w_{Ai} and w_{Bi} is shown on Fig. 5a. In the previous determination of the displacement of the shaft supports were seen as perfectly rigid, and in fact supports with rolling bearings are elastic. In elastic supports the nature of the elastic line is changed, the theoretical displacements and hence in particular the normal operation of the shaft and the whole operation of the machine are changed. The greater the elastic yield (displacement) of a bearing with a unit load, the less its stiffness. Stiffness of the bearing depends on its design, the shape of the rolling elements and others.

At bearing of misaligned shafts the bearing stiffness is of particular importance for determining of additional force F which loads the shafts.

After determination of the displacement of bearings w_{Ai} and w_{Bi}, the displacements in console part of both shafts w_i ($i = 1, 2$) can be determined. Displacements w_i are derived from the similarity of orthogonal triangles $\Delta A_iB_iB^I{}_i$ and $\Delta A_iC_iC^I{}_i$ (Fig. 5b):

$$\frac{l_i}{l_i + b_i + a} = \frac{w_{Ai} + w_{Bi}}{w_i + w_{Ai}}, \qquad \textbf{(15)}$$

whence it follows that:

$$w_i = \frac{(l_i + b_i + a).(w_{Ai} + w_{Bi})}{l_i} - w_{Ai}, \qquad \textbf{(16)}$$

where w_i is displacements of perfectly rigid shaft due to external force F depending on displacements in supports ($i = 1, 2$), m;

w_{Ai} и w_{Bi} – displacements in bearings (support A_i and support B_i) ($i = 1, 2$), m.

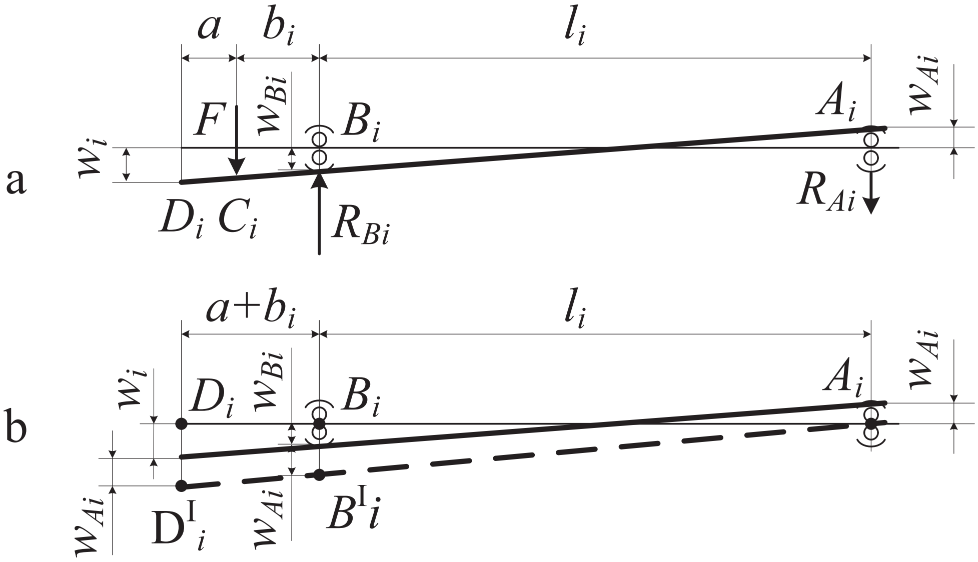

Figure 5: a – Displacement of perfectly rigid shaft depending on displacements in supports; b – scheme for geometrical determining of shaft displacement depending on bearings displacement

The displacement in bearings (w_{Ai} - for bearing in support A_i and w_{Bi} for bearing in support B_i) is a sum of initial clearance G_r and elastic deformations δ of roller bearing and rings.

$$w_{Ai} = G_{rAi} + \delta_{Ai}\ ,\ w_{Bi} = G_{rBi} + \delta_{Bi}\ , \qquad \textbf{(17)}$$

where G_{rAi} and G_{rBi} are initial radial clearance, for bearing in support A_i and for bearing in support B_i ($i = 1, 2$). Their values are taken from the catalog of roller bearings;

δ_{Ai}, δ_{Bi} – elastic deformations of the most loaded roller bodies and rings of the bearings, for bearing in support A_i and for bearing in support B_i, ($i = 1, 2$), N correspondingly.

The deformation δ of the most loaded ball (point contact) with force Q is equal of (Perel 1982, Sokolovski and al. 2013):

$$\delta_{Ai} = 2{,}79.10^{-6}.K.\left(Q_{Ai}^{2}.\rho_{\Sigma Ai}\right)^{\frac{1}{3}},$$

$$\delta_{Bi} = 2{,}79.10^{-6}.K.\left(Q_{Bi}^{2}.\rho_{\Sigma Bi}\right)^{\frac{1}{3}} \quad \textbf{(18)}$$

where δ_{Ai}, δ_{Bi} are deformations of the most loaded balls in supports A_i and B_i (i = 1, 2), m;

Q_{Ai}, Q_{Bi} – forces loaded the lowest-lying balls of bearing in supports A_i and B_i (i = 1, 2), N. They can be determined by the following formula (Kolesnikov and al. 2004):

$$Q_{Ai} = \frac{5R_{Ai}}{z_{Ai}},\ Q_{Bi} = \frac{5R_{Bi}}{z_{Bi}} \quad \textbf{(19)}$$

where R_{Ai}, R_{Bi} are support reactions of the shaft, for bearing in support A_i and for bearing in support B_i, (i = 1, 2), N. They can be determined from the dependences (Fig. 5a):

$$R_{Ai} = \frac{F(b_i - a)}{l_i};$$

$$R_{Bi} = \frac{F(b_i - a + l_i)}{l_i}. \quad \textbf{(20)}$$

where z_{Ai}, z_{Bi} are number of bearing balls in support A_i and in support B_i, (i = 1, 2);

K – parameter, characterizing the curvature of the contact surface, given in tables (Perel 1982, Sokolovski and al. 2013);

$\rho_{\Sigma Ai}$, $\rho_{\Sigma Bi}$ – the sums of curvature for bearings in support A_i and for bearing in support B_i (i = 1, 2), m^{-1}. They can be determined according to the form of contact surfaces (convex or concave) with a sphere (Perel 1982, Sokolovski and al. 2013).

For roller bearings (line contact) the deformation δ of the most loaded roll with force Q is [Л. Перель, 1982], (Sokolovski and al. 2013):

$$\delta_{Ai} = 1{,}1.10^{-10}\frac{Q_{Ai}^{0{,}925}}{l_{Ai}^{0{,}85}},$$

$$\delta_{Bi} = 1{,}1.10^{-10}\frac{Q_{Bi}^{0{,}925}}{l_{Bi}^{0{,}85}}, \quad \textbf{(21)}$$

where δ_{Ai}, δ_{Bi} are deformations of the most loaded roll in supports A_i and B_i (i = 1, 2), m;

Q_{Ai}, Q_{Bi} - forces loaded the lowest-lying balls of bearing in supports A_i and B_i (i = 1, 2), N. They can be determined by formula (18);

l_{Ai}, l_{Bi} – length of roll bearings in the supports A_i and B_i (i = 1, 2), m, given in catalogs for roller bearings.

Detailed definition of displacement in rolling bearings caused by elastic deformation of the rolling elements and rings is discussed in (Sokolovski and al. 2013).

The stiffness ratio $k_{o\Sigma}$ is obtained:

$$k_{o\Sigma} = \frac{f_{o\Sigma}}{F} \quad \textbf{(22)}$$

2.3. Determination of the elastic deformation of the coupling sleeve

The radial elastic displacements due to deformation of the coupling sleeve with sizes depending on the diameter d of the shaft are shown on Fig. 6. Elastic deformation of the coupling sleeve in console parts $z_{c\Sigma}$ is the sum of its displacement from the first shaft z_1 and displacement from the second shaft z_2, i.e.

$$z_{c\Sigma} = z_1 + z_2 \quad \textbf{(23)}$$

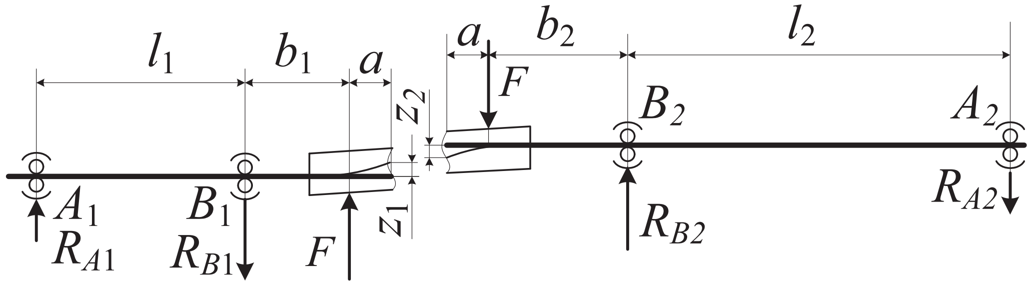

Figure 6: Displacement of coupling sleeve, located on the both console side of the shafts

Assuming that the coupling sleeve is a symmetrical part (Fig. 7a), then

$$z_{c\Sigma} = 2.z_i \quad \textbf{(24)}$$

Fig. 7 shows the displacement z_i of the console part of the coupling bush (i = 1,2). Displacement z_i is determined by the following dependence (Christov et al. 1980):

$$z_i = \frac{F}{6E.I}\left(3e^2.c - e^3\right), \quad \textbf{(25)}$$

where z_i is the displacement of the coupling sleeve in console part under the action of an additional force F, m;

F – additional force loading coupling sleeve, N;

E – elastic modulus for the sleeve material, Pa;

I_i - moment of inertia of the cross section of the coupling sleeve, m^4. It is known in the following formula:

$$I_i = \frac{\pi}{64}\left(D^4 - d^4\right), \quad \textbf{(26)}$$

where D, d are external and internal diameter of the coupling sleeve (Fig. 7), m;

a, e, c – distances according to Fig. 7, m.

The stiffness ratio $k_{c\Sigma}$ is received:

$$k_{c\Sigma} = \frac{f_{c\Sigma}}{F} \quad \textbf{(27)}$$

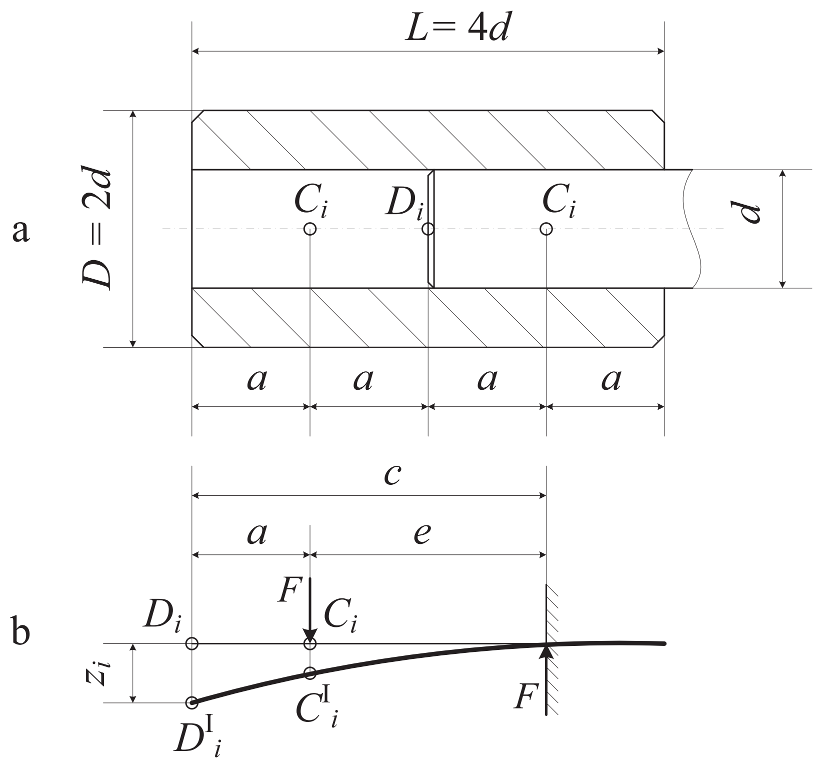

Figure 7: Displacement due to of the elastic deformation of the coupling sleeve

Fig. 8 shows the variation of the misalignment of two shafts, shafts displacement and coupling sleeve due to their elastic deformations under loading them with the additional force.

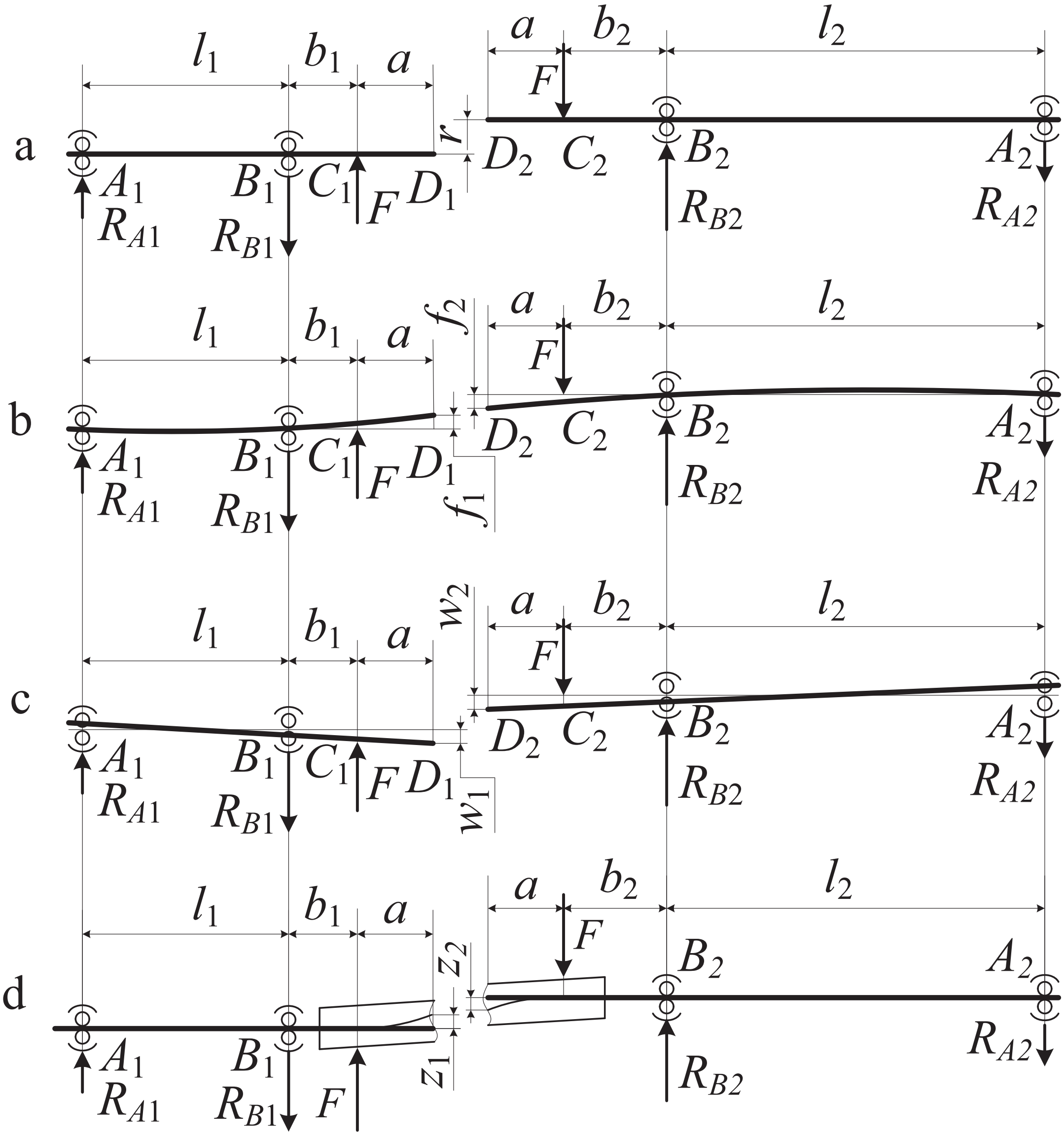

Figure 8: Types of deformation at connecting of radial displaced shafts: a – radial deviation of the shafts; b – elastic deformations of the shaft in the console parts; c – elastic deformation of the supports; d – elastic deformation of the coupling sleeve

CONCLUSIONS

The analysis to determine the dependence of the additional force loading shafts, supports and coupling elements shows that this effect can be reduced by:

- Reducing the deviations of shaft misalignment during assembly (reducing the numerator of the fraction in eq. 7);
- Selecting a coupling with higher compensating capability;
- Increasing the stiffness of the elements involved in the drive (increasing the numerator of the fraction in eq. 7).
- Best results will be obtained in the use of several methods for unloading

the driving elements from the additional force.

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