THE INTERNAL FORCES AND STRESSES IN THE BLADE OF A HORIZONTAL BANDSAW AFTER SOLVING THE STATICALLY INDETERMINATE PROBLEM

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ABSTRACT

The problem of the internal forces and stresses in the blade of a horizontal band-saw is statically indeterminate in the case of pre-tensioning by a spring. After our solution in a preceding article, in this paper the internal forces and stresses in the different parts of the blade are considered. Some remarks are addressed to the interpretation of such stresses in the bandsaw references reviewed. As well, stresses are analyzed relating to bending the blade over the wheel contours. Additional stresses are also outlined: at the guide rollers, other more bending stresses, as well as stresses due to torsion of the blade, etc.

Key words: horizontal bandsaw blade, spring mechanism, statically indeterminate system, internal forces, stresses

INTRODUCTION

In continuation of a preceding paper (Stefanov & Atanasov 2014), in this paper the internal forces and the stresses in a horizontal bandsaw blade are analyzed. In the case of tensioning the blade (the saw band) by a spring mechanism (Fig. 1), a statically indeterminate problem comes to be solved for determination of the tensile force X in the non-cutting blade's span (branch). This problem has been solved in (Stefanov & Atanasov 2014) by generalizing a solution from (Stefanov 2013). The following equation has been worked out:

$$X = \frac{1}{2d + \pi R + \dfrac{4Ebs}{c}} \left[-P\left(d_E + \frac{h_p}{2}\right) - \frac{PR}{\mu} - \frac{\pi R \Phi}{2} + 2Ebs\lambda_s - Ebs\Delta l_\theta - Ebsl\alpha_t \Delta t + \frac{2Ebs\Phi}{c} \right] \quad \textbf{(1)}$$

where

$$\mu = \operatorname{tg}\alpha = \frac{1}{\pi}\ln\left(1 + \frac{P}{X - \Phi/2}\right) \quad \textbf{(2)}$$

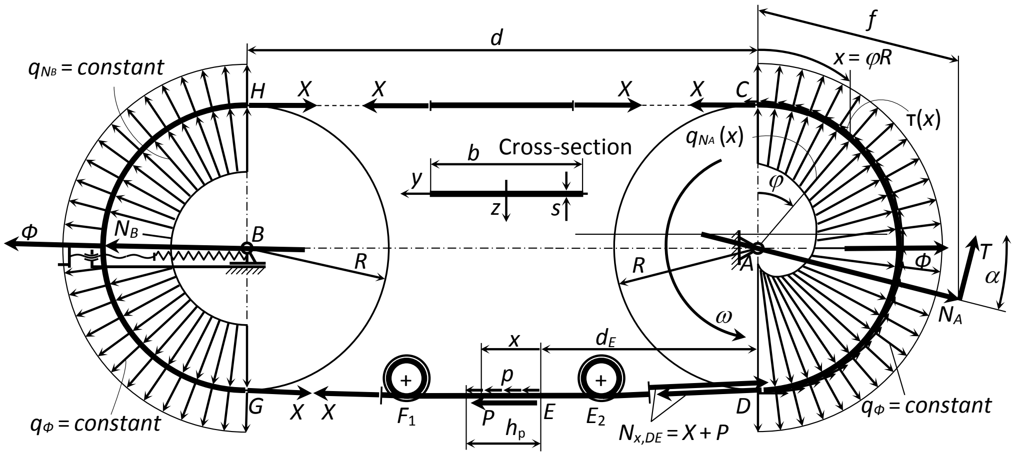

Figure 1: The blade together with its internal tensile forces

The nomenclature of the symbols in these equations follows (see also Fig. 1).

E – Young's modulus of the blade material;

λ_s – pull-out (as a distance) of the left spring's end for pre-tensioning the blade;

C – spring's stiffness (spring's constant);

R, d, d_E, h_p, α – dimensions and angle as shown in Fig. 1;

L – length of the blade ($l = 2d + 2\pi R$);

Δl_θ – lengthening of l along the central blade fibre as a result from driven (left) wheel tilt, or both from this tilt and driving wheel tilt if any; if the tilt causes shortening of l, then Δl_θ is taken with negative algebraic value;

Δt – heating the blade (in ^{o}C);

α_t – coefficient of free thermal expansion per 1^{o}C;

μ – working coefficient of friction (coherence) with the driving wheel ($\mu = \mathrm{tg}\alpha$);

P – cutting force, longitudinal to the blade;

Φ – force of mass inertia of each of the two blade half-circumferential parts ($\Phi = 2\rho bsv^2$ where $v = \omega R$ is the blade speed and ρ is the mass density).

Eqs. (1) and (2) are worked out after expanding the following deformation compatibility equation (Stefanov & Atanasov 2014):

$$\mathbf{2\lambda_s = \Delta l + \Delta l_\theta + \Delta l_t + 2\delta} \qquad \mathbf{(3)}$$

where Δl the elastic lengthening of l caused by blade loading, $\Delta l_t = \alpha_t l \Delta t$ is free thermal lengthening of l, and δ is the spring lengthening. The deformation compatibility according to Eq. (3) can be explained in the following shortest way: the sum of the lengthenings Δl, Δl_θ, Δl_t and 2δ covers $2\lambda_s$. Any different try for determination of X, as well as of the tensile forces N_x in the blade sections, will be incorrect, if the compatibility Eq. (3) is not fulfilled.

Eqs. (1) and (2) remain valid also for a vertical bandsaw where λ_s is a push-up (as a distance) of the bottom end of the spring (or a pull-up of the top spring's end).

The X force cannot be directly computed by Eq. (1) since it participates also in the right-hand side via μ, behind the 'ln' sign in Eq. (2). Instead, Eq. (1), so presented, allows practical computation of X by means of fixed-point iteration (Stefanov 2013) of other method of successive approximations. This can be programmed e.g. in an Excel file giving automatically X and μ to the user. Subsequently, the following parameters can be computed: the forces $N_B = 2X - \Phi$ and $N_A = (P + N_B)\cos\alpha$ where $\alpha = \mathrm{arctg}\mu$, the traction force (from the friction) $T = (P + N_B)\sin\alpha$, as well as its moment arm $f = PR/T$ ($\neq R$). The origin of these equations (from the blade kinetostatic equilibrium conditions) can be seen in (Stefanov & Atanasov 2014). There, the reasons are also given that the forces N_B, N_A and Φ are practically horizontal.

It is to interpose here, by the way, that if N_B had been exerted by a lever-weight or hydraulic mechanism, this force would have assumed the role of a given, constant and *independent* load. Then, $X = (N_B + \Phi)/2$. Eq. (2) remains valid, as well as $\alpha = \mathrm{arctg}\mu$, $N_A = (P + N_B)\cos\alpha$, $T = (P + N_B)\sin\alpha$ and $f = PR/T$. *In that case, the problem of the tensile internal forces and stresses in a saw band becomes statically determinate* and much easier: *neither assembly stresses nor thermal stresses appear in statically determinate systems* under any dictates of any compatibility conditions (Stefanov 2007). On this occasion, it is not correct to introduce blade thermal forces and stresses in the bandsaw books without clarifying that they are absent in case of hydraulic or lever-weight tensioning the blade.

1 THE TENSILE FORCE $X \equiv X_0$ IN THE CASE OF $P = 0$

The case of $P = 0$ means idle running of the band (no cutting). According to Eq. (2), $\mu = \mathrm{tg}\alpha = 0$: the friction force T in Fig. 1 is absent, and the N_A force takes a (practically) horizontal position with a magnitude $N_A \equiv N_{A,0} = N_B \equiv N_{B,0} = 2X_0 - \Phi$. The same tensile force $N_x = constant = X \equiv X_0$ settles along the whole band's length. It will be calculated using Eq. (1) after setting $P = 0$ there. However, it is to take into consideration that the addend PR/μ in the brackets transforms into the indeterminate form [0/0]. For revealing it, the L'Hôpital's rule applies (Stefanov 2013). After certain mathematical manipulations, we have

$$X_0 = \frac{1}{l + \dfrac{4Ebs}{c}}\left[2Ebs\lambda_s - Ebs\Delta l_\theta - Ebsl\alpha_t\Delta t + \frac{2Ebs\Phi}{c}\right] \qquad \textbf{(4)}$$

This equation can be represented in the form $X_0 = X_\mathrm{M} - X_\theta - X_t + X_\Phi$ with meanings of the addends, as follows. With $\Delta t = 0$, $\Delta l_\theta = 0$ and $\Phi = 0$, Eq. (4) yields the assembly (mounting) tension (pre-tension): $X_\mathrm{M} = 2Ebs\lambda_s/(l + 4Ebs/c)$. Hence, the necessary value of λ_s for enabling desired pre-tension X_M is $\lambda_s = X_\mathrm{M}(l + 4Ebs/c)/(2Ebs)$. After wheel tilting, Δl_θ appears and $X_\theta = Ebs\Delta l_\theta/(l + 4Ebs/c)$ is subtracted from X_M. If $\Delta l_\theta < 0$ (shortening instead of lengthening the blade), X_θ will not be a subtrahend but addend to X_M. With starting the engine and appearing Φ, $X_\Phi = 2Ebs\Phi/(cl + 4Ebs)$ is added to X_M. And with appearance of $\Delta t > 0$ (heating), $X_t = Ebsl\alpha_t\Delta t/(l + 4Ebs/c)$ is subtracted from X_M.

In the borderline case of $c \to \infty$, the equations $X_\mathrm{M} = 2Ebs\lambda_s/l$, $X_\theta = Ebs\Delta l_\theta/l$, $X_\Phi = 0$ and $X_t = Ebs\alpha_t\Delta t$ take place. In other words, in case of a very stiff spring, the appearing centrifugal forces while setting the band in motion do not influence the pre-tension which is $X_\mathrm{M} \approx 2Ebs\lambda_s/l$. This means, with lengthening the band by the centrifugal forces and supervened loosing the spring, the intensity q_{NB} decreases by as much as the appearing intensity q_Φ is. In addition, the heating Δt leads to a decrease (a subtrahend) X_t of X_M, and this X_t converges to a maximum value $Ebs\alpha_t\Delta t$. X_θ also converges to an extremum value $Ebs\Delta l_\theta/l$.

Let the other borderline case of $c \to 0$ (of an infinitely soft spring which necessitates $\lambda_s \to \infty$) be also studied. For this purpose, it is pertinent to represent Eq. (7) in this form:

$$X_0 = \frac{1}{cl + 4Ebs}\left[2cEbs\lambda_s - cEbs\Delta l_\theta - cEbsl\alpha_t\Delta t + 2Ebs\Phi\right]. \qquad \textbf{(5)}$$

Now, $X_\mathrm{M} \to c\lambda_s/2$, $X_\theta = 0$, $X_t = 0$ and $X_\Phi = \Phi/2$. This means that with a very soft spring the appearing centrifugal forces increase the pre-tension by an addition which converges to a maximum value $\Phi/2 = \rho bsv^2$. And, the subtrahends due to the tilting and heating drop out. Thus, it turns out that for keeping the pre-tension without weakening influence of the tilting and heating, and for strengthening this tension by the centrifugal forces, a soft spring is more appropriate than a stiff one. Then, the system converges to statical determinacy like with using a lever-weight or hydraulic mechanism.

2 BLADE STRESSES RESULTING FROM $N_X(X)$

The expressions of the internal tensile (normal) force $N_x(x)$ along the blade sections are:

$$CD:\ N_x(x) \equiv N_{x,CD}(x) = e^{\mu x/R}(X - \Phi/2) + \Phi/2 \quad \textbf{(6)}$$

$$DE:\ N_x \equiv N_{x,DE} = constant = X + P \quad \textbf{(7)}$$

$$EF:\ N_x(x) \equiv N_{x,EF}(x) = X + (P/h_\mathrm{p})(h_\mathrm{p} - x) \quad \textbf{(8)}$$

$$FGHC:\ N_x \equiv N_{x,FGHC} = constant = X \quad \textbf{(9)}$$

Eqs. (7) and (9) are understandable from Fig. 1, and the origin of Eqs. (6) and (8) can be seen in (Stefanov & Atanasov 2014). In each cross-section along each blade section, the stress resulting from N_x is in the form of $\sigma_{Nx} = N_x/(bs)$. Correspondingly, $\sigma_{Nx,CD} = N_{x,CD}/(bs)$, $\sigma_{Nx,DE} = N_{x,DE}/(bs) = X/(bs) + P/(bs)$ (the greatest stress σ_{Nx}), $\sigma_{Nx,EF} = N_{x,EF}/(bs)$ and $\sigma_{Nx,FGHC} = N_{x,FGHC}/(bs) = X/(bs)$.

According to (Atanasov 2012), other authors introduce stress from the cutting force as $\sigma_P = P/(bs)$. The present papers shows that there is not such an individual stress: none of the above four internal forces N_x is equal to P. On this occasion, two remarks follow.

The first one is that any external longitudinal force is not supposed to be divided by a cross-sectional area, arbitrarily, and the result is not supposed to be treated as a stress resulting from that force. It is to recollect from Strength of Materials (Stefanov 1989 and 2007) that every stress in a cross-section acts on an infinitesimal face with a defined location in the cross-section. That stress is determined by an equation that does not involve any external load but the respective internal load in the cross-section. In the case in hand, the simplest equation for a stress in valid as $\sigma_{Nx} = N_x/(bs)$; only in such a case the stress is considered the same on all the infinitesimal faces of the cross-section.

The second remark is that introducing and considering a separate stress $\sigma_P = P/(bs)$ is unnecessary and confusing. It necessitates additional explanation that $\sigma_P = P/(bs)$ is one of the two components of $\sigma_{Nx,DE} = X/(bs) + P/(bs)$, and that namely this component is missing in any of the other stresses $\sigma_{Nx,CD}$, $\sigma_{Nx,EF}$ and $\sigma_{Nx,FGHC}$.

In the studied papers and books, again confusing another stress is introduced as a stress from the traction force. The latter is defined as "equal to the difference between the tensile forces in the cutting and non-cutting span". That difference is divided by bs and the result is considered to be a stress from the traction force. However, the tensile forces in the cutting span are three: according to Eqs. (7), (8) and (9). Which of them will be taken for building that "difference" and why? If the difference between $N_{x,DE} = = X + P$ and $N_{x,HC} = N_{x,FGHC} = X$ is meant, it is equal to P. Respectively, it will involve again the above stress $\sigma_P = P/(bs)$.

Should a traction force different from P be discussed, then such one is $T = (P + N_B)\sin\alpha = (P + 2X - \Phi)\sin\alpha$ in Fig. 1. Besides, it is imaginarily concentrated: it results from the actual longitudinally distributed force along the half-circumference with an intensity $\tau(x) = \mu q_{N_A}(x) = \mu N_A(x)/(2R)$. That imaginary force T even does not act on the blade but stays at the distance (moment arm) f from A. Thus, it is not proper to introduce a stress $\sigma_T = T/(bs)$ since there is not such one; there is $\sigma_{Nx,CD}(x) = N_{x,CD}(x)/(bs)$ where $N_{x,CD}$ contains implicitly $\tau(x)$ by Eq. (6).

Next formula, popular but incorrectly introduced in studied references, is for a separately considered stress resulting from the heating as $\sigma_t = \alpha_t E\Delta t$. On this occasion, let the expanded equation of e.g. $\sigma_{Nx,FGHC} = N_{x,FGHC}/(bs) = X/(bs)$ be written:

$$\sigma_{Nx,FGHC} = \frac{1}{2d + \pi R + \dfrac{4Ebs}{c}}\left[-\frac{P}{bs}\left(d_E + \frac{h_p}{2}\right) - \frac{PR}{\mu bs} - \frac{\pi R\Phi}{2bs} + 2E\lambda_s - E\Delta l_\theta - El\alpha_t\Delta t + \frac{2E\Phi}{c}\right] \quad \textbf{(10)}$$

It is obvious that „$\sigma_t = \alpha_t E\Delta t$" has something in common with, but is not the component $-El\alpha_t\Delta t/(2d + \pi R + 4Ebs/c)$ of $\sigma_{Nx,FGHC}$. Only in case $P = 0$ when $\sigma_{Nx,FGHC}$ becomes $\sigma_{Nx,FGHC,0} = X_0/(bs)$, and only if $c \to \infty$, Eq. (4) yields $\sigma_t = X_t/(bs) = -\alpha_t E\Delta t$, with minus sign, by the way. It is incorrect and unnecessary to give „$\sigma_t = \alpha_t E\Delta t$" without the validity conditions revealed here, and separated from the other components of $\sigma_{Nx} = N_x/(bs)$. It is especially important to emphasize that a stress component „$\sigma_t = \alpha_t E\Delta t$" is missing in every σ_{Nx} in case $c \to 0$, as well as in case of hydraulic or lever-weight tensioning the blade.

In papers studied, again separately a stress of the type $\sigma_\theta = X_\theta/(bs) = E\Delta l_\theta/l$ is introduced. It is analogous to „$\sigma_t = \alpha_t E\Delta t$", and thus the above remarks are valid again.

In addition, Eq. (4), together with $X_\Phi = \Phi/2 = \rho bsv^2$, only in case $P = 0$ and $c \to 0$, yields a stress $\sigma_\Phi = X_\Phi/(bs) = \rho v^2$, again as a component of $\sigma_{Nx,FGHC,0}$. Whereas, in studied references, an individual equation is given as $\sigma_\Phi = 0{,}01v^2\gamma/g = 0{,}01\rho v^2$ (the multiplier 0,01 appears when v, γ and g take values from an old measurement system). For such a formula, unnecessarily separated from the other components of a stress of the type $\sigma_{Nx} = N_x/(bs)$, again additional explanations are needed: it is valid in case $P = 0$ and $c \to 0$, also in case $P = 0$ and the blade is tensioned by hydraulic or lever-weight mechanism. Especially in case $c \to \infty$ it is important to emphasize that a stress component in the form of $\sigma_\Phi = \rho v^2$ is absent.

3 BLADE STRESSES RESULTING FROM BENDING WITH M_Y

There is, in fact, a second statically indeterminate internal load, as well: the bending moment M_y due to the bending (the deflection) of the saw band along the two half-circumferences. In the case considered, the M_y moment is, however, determinable based on another compatibility condition: $M_y = EJ_y/\rho$ (Stefanov 2007) where ρ is the radius of the deflection curvature. From the idealization for exact half-circumferences with $\rho = R = constant$, $M_y = constant$ results.

This bending moment is very little: a saw band is (easily) flexible, though. However, it causes a significant stress of bending $\sigma_{be} = M_y/W_y$ since the cross-sectional moment of resistance $W_y = bs^2/6$ is also very little: $\sigma_{be} = M_y/W_y = [E(bs^3/12)/R]/(bs^2/6) = Es/(2R)$. In a calculation example mentioned in (Stefanov 2013), $\sigma_{be} = 250$ MPa was obtained. This is a high stress, and it will be greater at a horizontal bandsaw for the smaller radius R. Thus, even only σ_{be} already requires manufacturing the saw bands of complex alloyed and other steels of high ultimate strength in the order of 1000 and more MPa.

At this point, a discussion should take place regarding the well-known practical rule $s \le 2R/1000$ (Atanasov 2012). The origin of this rule is, by the way, not always well-known. It results, in fact, from the strength condition $\sigma_{be} = Es/(2R) \le \sigma_{adm}$ with $E = 2.10^{11}$ generally for steels and admissible stress σ_{adm} set to be 200 MPa. Indeed, from $2.10^{11}s/(2R) \le 200.10^6$, $s \le 2R/1000$ results. However, the horizontal bandsaw wheels need smaller R. Then, the rule $s \le 2R/1000$ would require smaller s, i.e. use of thinner blades. On the other hand, horizontal bandsaws should be able to cut using blades like the vertical ones in order to achieve equivalent operational characteristics.

Hence, the rule $s \leq 2R/1000$ is not valid for a horizontal bandsaw. For example, in case the radius R is twice as small as of a vertical bandsaw, then the same rule will give the same thickness s if modified into $s \leq 2R/500$. This means $\sigma_{\text{adm}} = 400$ MPa instead of $\sigma_{\text{adm}} = 200$ MPa, i.e. the blade steal for a horizontal bandsaw should be of higher strength than for a vertical bandsaw. Actually, instead of introducing any rule for s selection, it is more correct to state, for all kinds of bandsaws, the strength condition $Es/(2R) \leq \sigma_{\text{доп}}$ solved for s: $s \leq 2R\sigma_{\text{adm}}/E$.

Besides, it should be clear that the written strength condition is valid for only taking into account $\sigma_{\text{be}} \equiv \sigma_{\text{be},My,CD} = \sigma_{\text{be},My,GH}$ without involving σ_{Nx}. While σ_{Nx} is the same at all the cross-section points, σ_{be} acts only at $z_{\max} = s/2$ (at the bottommost point) and at $z_{\min} = -s/2$ (at the topmost point). Thus, next step for strength calculation is forming the resultant normal stress $\max\sigma_{My,Nx} = \sigma_{\text{be}} + \sigma_{Nx} = Es/(2R) + \sigma_{Nx}$ at the bottommost or topmost point. The question arises, in which cross-section? Since $\max\sigma_{Nx} = \sigma_{Nx,DE} = \sigma_{Nx,D}$, then the cross-section D has to bear $\max\sigma_{My,Nx} = Es/(2R) + \sigma_{Nx,D} = Es/(2R) + X/(bs) + P/(bs)$.

4 OTHER MORE STRESSES

At the blade guides, e.g. at F_1 (Fig .1), another bending stress appears: $\sigma_{\text{be}} \equiv \sigma_{\text{be},My,F1} = Es/(2\rho_{F1})$ where ρ_{F1} is the smallest curvature radius of blade deflection round the guide roller.

Due to the wheel tilt, according to Fig. 3 in (Stefanov & Atanasov), stresses of a second bending appear, now with M_z. Besides, according to the same Fig. 3, the tilt angle θ transforms into an angle of deformational turn (angular displacement) of a corresponding cross-section around its x axis. Yet this means blade torsion with a torsional moment M_t: an essential fact to which nearly no due attention is paid in the studies reviewed. Respectively, shear stresses of the type $\tau_t = M_t/W_t$ appear where W_t is the moment of resistance to torsion of a rectangular cross-section (Stefanov 1989).

If the driven or both wheels have a crowned rim, "bending of the very rectangular cross-section" (as a thin plate) occurs over it. Thus, a normal stress σ_y also appears together with σ_x. Formulae of thin-plate bending theory can be applied as done e.g. in (Eschler 1982).

CONCLUSION

The stresses considered are nominal (remote), i.e. the basic ones according to the Strength of Materials course. It was important to clear them and combine them correctly as stress components. And, before that, it was important to clear the internal forces after solving the statically indeterminate problem. In general, an exacter picture has been presented regarding all the possible nominal stresses and their correct combination equations for strength calculation done in the way as the Strength of Materials science requires. Essential differences in stress incidence have been emphasized in comparison to hydraulic or lever-weight tensioning the blade.

Besides traditional strength calculations of nominal stresses, nowadays all kinds of stresses, including concentrated local ones, are practically determined. This is enabled by means of the finite-element (FE) method. Contemporary FE software, e.g. Autodesk Inventor FE environment, already makes pointless many previous studies on internal forces and stresses. Our ambitions are to enable later entire 3D FE modeling associated with Fig. 1. This will serve for verification of the postulations and the equations in this paper, for automatic simulation of all the stresses, including those from the above chapter 3, etc. In this regard, a solid foundation has already been laid in (Stefanov & Scholz 2014). Besides, FE experience from (Atanasov et al. 2012) will be further developed.

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