

## MODELLING OF THE ENERGY NEEDED FOR DEFROSTING OF WOOD MATERIALS IN THE HYGROSCOPIC RANGE

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### ABSTRACT

A mathematical model of the specific heat energy, which is needed for defrosting of wood materials in the hygroscopic range,  $q_{\text{dfr}}$ , has been suggested. The specific heat energy  $q_{\text{dfr}}$  consist of two parts: the energy needed for heating of the frozen wood until melting of the ice in it,  $q_{\text{wfr}}$ , and the energy needed for the melting of the ice formed from the bound water in the wood,  $q_{\text{bw}}$ . Using the suggested model computations have been carried out for the determination of  $q_{\text{wfr}}$ ,  $q_{\text{bw}}$ , and  $q_{\text{dfr}} = q_{\text{wfr}} + q_{\text{bw}}$  during defrosting of oak, pine, beech, and poplar frozen wood with moisture content from  $0.2 \text{ kg}\cdot\text{kg}^{-1}$  up to the border of the hygroscopic diapason and with initial temperature in the range from  $-20 \text{ }^{\circ}\text{C}$  to  $-2 \text{ }^{\circ}\text{C}$ , at which the thawing of the ice formed from the bounded water is completed.

**Key words:** modelling, wood defrosting, specific heat energy, wood specie, hygroscopic range

### INTRODUCTION

When sizing the power of the sources of heat energy, which are used for the supply of the equipment for defrosting of wood materials, it is necessary to take into consideration the need for energy both for the heating of the frozen wood and for the thawing of the ice in it during the winter (Shubin 1990, Pervan 2000, Trebula and Klement 2003, Videlov 2003).

In Deliiski (2003, 2004, 2011) 3-, 2-, and 1-dimensional models have been created, solved, and verified of the transient non-linear heat conduction and energy consumption in frozen wood materials with prismatic and cylindrical shape during their thermal treatment. The solution of these models, in which the mechanism for distribution of the temperature in the wood materials is described by rather complex differential equations with partial derivatives, is carried out with the help of specialized software, developed by the author.

For the calculation of the need of thermal energy for the heating of the frozen wood and melting of the ice in it with the help of non-stationary mathematical models it is necessary to have the mentioned specialized software, whose accessibility however is very limited.

The aim of the present work is to suggest an easy for engineering applications mathematical model of the specific heat energy consumption,  $q_{\text{dfr}}$ , which is needed for defrosting of frozen wood materials in the hygroscopic range. For the achieving of this goal mathematical models of the two consisting parts of  $q_{\text{dfr}}$  have been also suggested, as follows: of the energy needed for heating of the frozen wood until melting of the ice formed from the bounded water in it,  $q_{\text{wfr}}$ , and of the energy needed for melting of the mentioned ice,  $q_{\text{bw}}$ .

# 1. MODEL OF THE ENERGY NEEDED FOR HEATING OF THE FROZEN WOOD UNTIL MELTING OF THE ICE IN IT IN THE HYGROSCOPIC RANGE

$$q_{\text{wfr}} = \frac{\rho_w c_{\text{wfr}} (T_{\text{dfr}} - T_{\text{w0}})}{3.6 \cdot 10^6} @ u_{\text{nfw}} < u \leq u_{\text{fsp}}^{271.15} \ \& \ T_{\text{w0}} < T_{\text{dfr}}, \quad (1)$$

where  $q_{\text{wfr}}$  is the specific heat energy needed for heating of frozen wood until melting of the ice in it, kWh·m<sup>-3</sup>;

$\rho_w$  – density of the frozen wood in the hygroscopic range, kg·m<sup>-3</sup>;

$c_{\text{wfr}}$  – specific heat capacity of the frozen wood, J·kg<sup>-1</sup>·K<sup>-1</sup>;

$T_{\text{w0}}$  – temperature of the frozen wood at the beginning of its defrosting, K;

$T_{\text{dfr}}$  – wood defrosting temperature, i.e. the temperature at which the ice formed from the bounded water in the wood is transformed completely into a liquid state, K. The temperature  $T_{\text{dfr}}$  is determined according to equation (6) given below when  $0.12 \text{ kg} \cdot \text{kg}^{-1} \leq u < u_{\text{fsp}}^{271.15}$  or according to equation (7) if  $u \geq u_{\text{fsp}}^{271.15}$ ;

$u$  – wood moisture content of the frozen wood, kg·kg<sup>-1</sup>;

$u_{\text{fsp}}$  – fiber saturation point of a given wood specie, kg·kg<sup>-1</sup>. The value of  $u_{\text{fsp}}$  is determined according to a suggested by Stamm (1964) equation, given in Deliiski (2013);

$$\rho_w = \rho_b \frac{1 + u}{1 - \frac{S_v}{100} [u_{\text{fsp}}^{293.15} - 0.001(T_{\text{dfr}} - 293.15) - u]} @ u_{\text{nfw}} < u \leq u_{\text{fsp}}^{271.15} \ \& \ T \leq T_{\text{dfr}}, \quad (2)$$

$$c_{\text{wfr}} = K_c \frac{526 + 2.95 \left( \frac{T_{\text{w0}} + T_{\text{dfr}}}{2} \right) + 0.0022 \left( \frac{T_{\text{w0}} + T_{\text{dfr}}}{2} \right)^2 + 2261u + 1976u_{\text{nfw}}}{1 + u}, \quad (3)$$

The specific energy needed for heating of frozen wood until melting of the ice in it in the hygroscopic range,  $q_{\text{wfr}}$ , can be determined according to equation given in Deliiski (2003):

$u_{\text{fsp}}^{271.15}$  – fiber saturation point of the wood at temperature  $T = 271.15$  K, i.e. at  $t = -2$  °C, at which the thawing of the ice formed from the bounded water is completed (Chudinov 1968), kg·kg<sup>-1</sup>. The values of  $u_{\text{fsp}}^{271.15}$  are determined according to the equation given in Deliiski (2013);

$u_{\text{nfw}}$  – content of non-frozen water in the wood at given temperature  $T_{\text{w0}} \leq 271.15$  K, kg·kg<sup>-1</sup>. The value of  $u_{\text{nfw}}$  is determined according to equation (5) given below.

The multiplier  $3.6 \cdot 10^6$  in the denominator of equation (1) ensures that the values of  $q_{\text{wfr}}$  are obtained in kWh·m<sup>-3</sup> instead of in J·m<sup>-3</sup>.

For practical usage of equation (1) for the determination of  $q_{\text{wfr}}$  it is needed to have mathematical descriptions of  $\rho_w$  and  $c_{\text{wfr}}$  in the hygroscopic range. In Deliiski (2013) the following descriptions of  $\rho_w$  and  $c_{\text{wfr}}$  in the hygroscopic range have been suggested:

$$K_c = 1.06 + 0.04u + \frac{0.00075 \left( \frac{T_{w0} + T_{dfr}}{2} - 271.15 \right)}{u_{nfw}}, \quad (4)$$

$$u_{nfw} = 0.12 + \left( u_{fsp}^{293.15} - 0.001T_{dfr} + 0.17315 \right) \exp[0.0567(T_{w0} - 271.15)], \quad (5)$$

$$T_{dfr} = 271.15 + \frac{\ln \frac{u_{nfw} - 0.12}{u_{fsp} - 0.12}}{0.0567} \quad @ \quad 0.12 \text{ kg} \cdot \text{kg}^{-1} \leq u = u_{nfw} < u_{fsp}^{271.15}, \quad (6)$$

$$T_{dfr} = 271.15 \quad @ \quad u \geq u_{fsp}^{271.15}, \quad (7)$$

where  $u_{fsp}^{293.15}$  is the fiber saturation point of the wood at temperature  $T = 293.15$  K, i.e. at  $t = 20$  °C, whose values for separate wood species are given in the specialized literature,  $\text{kg} \cdot \text{kg}^{-1}$ ;

$\rho_b$  – basic density of the wood given in the specialized literature,  $\text{kg} \cdot \text{m}^{-3}$ ;

$S_v$  – volume shrinkage of a given wood specie given in the specialized literature, % .

The calculated according to equations (2) to (6) change in  $\rho_w(\rho_b, u, u_{fsp}^{293.15}, S_v)$  and in  $c_{wfr}(u, u_{fsp}^{293.15}, T_{w0})$  for oak, beech, pine and poplar wood with  $t_{w0} = -20$  °C in the range from  $u = 0.20 \text{ kg} \cdot \text{kg}^{-1}$  to  $u = u_{fsp}^{271.15}$  are shown on Fig. 1.

The calculated according to equation (1) change in  $q_{wfr} = f(u)$  for the studied four wood species with  $t_{w0} = -10$  °C and  $t_{w0} = -20$  °C are shown on Fig. 2.

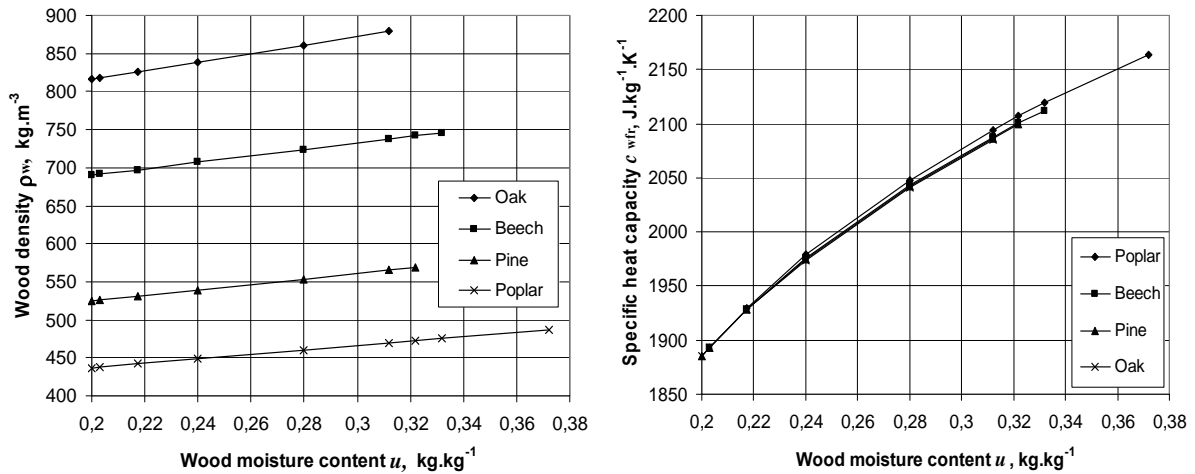


Figure 1: Change in  $\rho_w$  (left) and in  $c_{wfr}$  (right) during defrosting of wood with  $t_{w0} = -20$  °C depending on  $u$  and on the wood specie

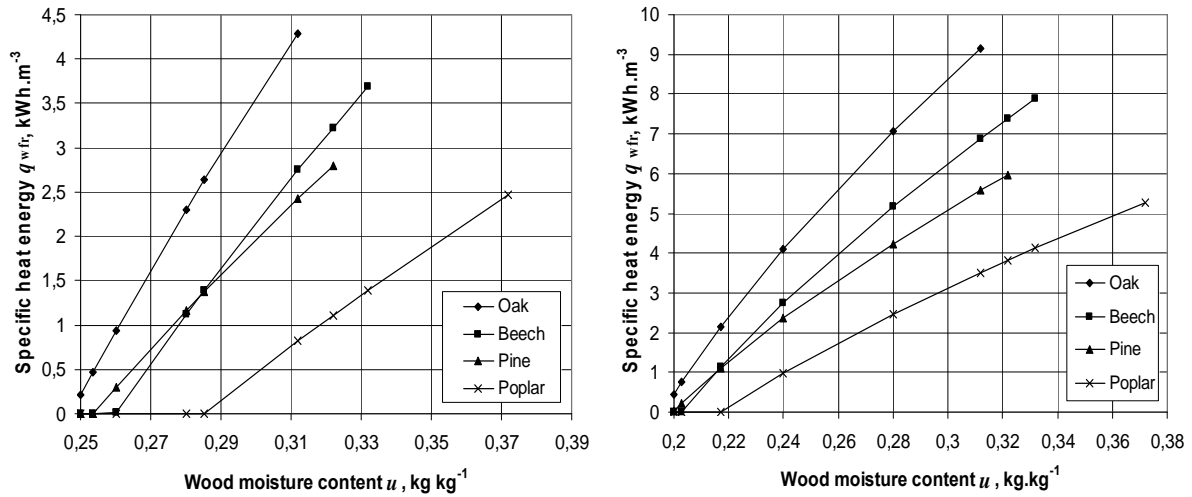


Figure 2: Change in  $q_{wfr}$  during defrosting of wood with  $t_{w0} = -10$  °C (left) and with  $t_{w0} = -20$  °C (right) depending on  $u$  and on the wood specie

## 2. MODEL OF THE ENERGY NEEDED FOR MELTING OF THE ICE IN FROZEN WOOD IN THE HYGROSCOPIC RANGE

The specific energy needed for melting of the ice formed from the freezing of

$$q_{bw} = \frac{\rho_w c_{bw} (T_{dfr} - T_{w0})}{3.6 \cdot 10^6} @ u_{nfw} < u \leq u_{fsp} \ \& \ T_{w0} < 271.15 \text{ K}, \quad (8)$$

where  $q_{bw}$  is the specific heat energy needed for melting of the ice, formed from the bounded water in the wood,  $\text{kWh.m}^{-3}$ ;

$c_{bw}$  – specific heat capacity of the ice formed from the freezing of the bounded water in the wood, whose values (in  $\text{J.kg}^{-1}$

$$c_{bw} = 1.8938 \cdot 10^4 \left( u_{fsp}^{293.15} - 0.001 T_{dfr} - 0.17315 \right) \frac{\exp \left[ 0.0567 \left( \frac{T_{w0} + T_{dfr}}{2} - 271.15 \right) \right]}{1 + u}. \quad (9)$$

The meaning of the other variables involved in equations (8) and (9) are given above when clarifying equations (1) to (7). The calculated according to equation (9) change in  $c_{bw}$  for the studied four wood species with  $t_{w0} = -10$  °C and  $t_{w0} = -20$  °C depending on  $u$  is shown on Fig. 3.

bounded water in the wood,  $q_{bw}$ , can be determined according to the equation given in Deliiski (2013b)

$^1 \cdot \text{K}^{-1}$ ) in the ranges  $u_{nfw} < u \leq u_{fsp}$  &  $T \leq 271.15 \text{ K}$  can be calculated according to the following equation, given in Deliiski (2013):

The calculated according to equation (8) change in  $q_{bw}$ , using also equations (2) and (8), for frozen wood with  $t_{w0} = -10$  °C and  $t_{w0} = -20$  °C depending on  $u$  is shown on Fig. 4.

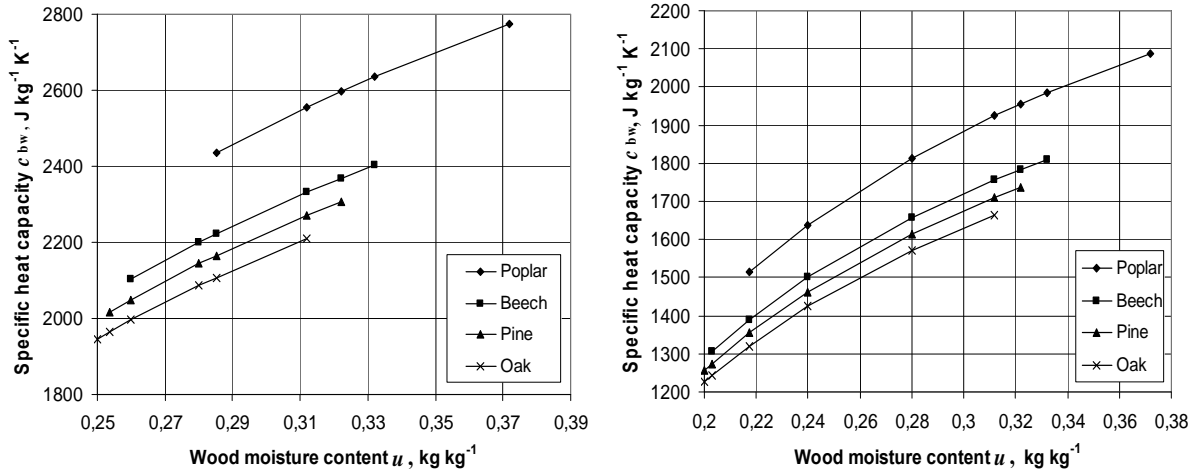


Figure 3: Change in  $c_{bw}$  during defrosting of wood with  $t_{w0} = -10 \text{ °C}$  (left) and with  $t_{w0} = -20 \text{ °C}$  (right) depending on  $u$  and on the wood species

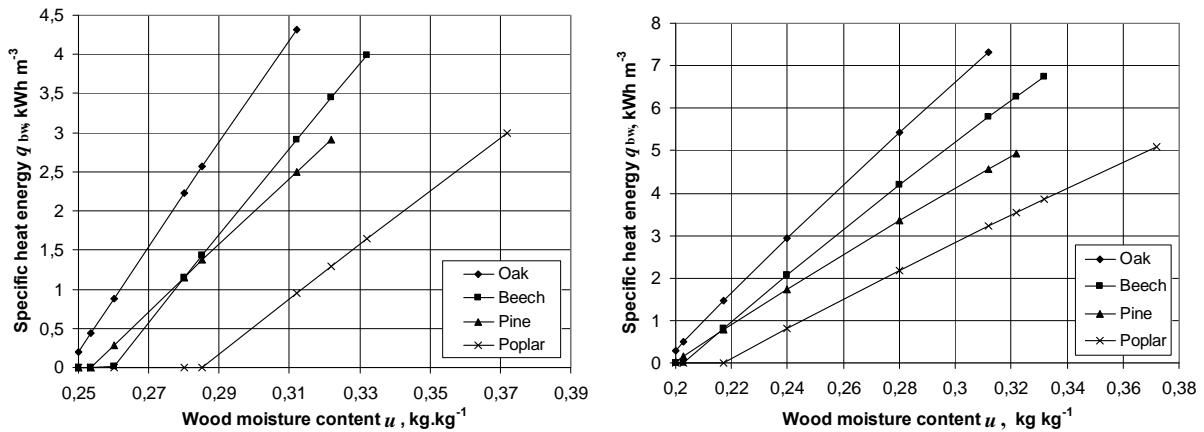


Figure 4: Change in  $q_{bw}$  during defrosting of wood with  $t_{w0} = -10 \text{ °C}$  (left) and with  $t_{w0} = -20 \text{ °C}$  (right) depending on  $u$  and on the wood species

### 3. MODEL OF THE ENERGY NEEDED FOR DEFROSTING OF WOOD MATERIALS IN THE HYGROSCOPIC RANGE

The specific energy, which is needed for defrosting of the wood in the hygroscopic range,  $q_{dfr}$ , is equal to the sum of the energy needed for heating of frozen wood until melting of the ice in it in the hygroscopic diapason,  $q_{wfr}$ , and of the energy needed for the melting of the ice

formed in the wood from the freezing of the bound water in it,  $q_{bw}$ , i.e.  $q_{dfr}$  can be determined according to the equation

$$q_{dfr} = q_{wfr} + q_{bw} \quad (10)$$

The calculated according to equations (10), (1), and (8) change in  $q_{dfr}$  for the studied four wood species with  $t_{w0} = -10 \text{ °C}$  and  $t_{w0} = -20 \text{ °C}$  depending on  $u$  is shown on Fig. 5.

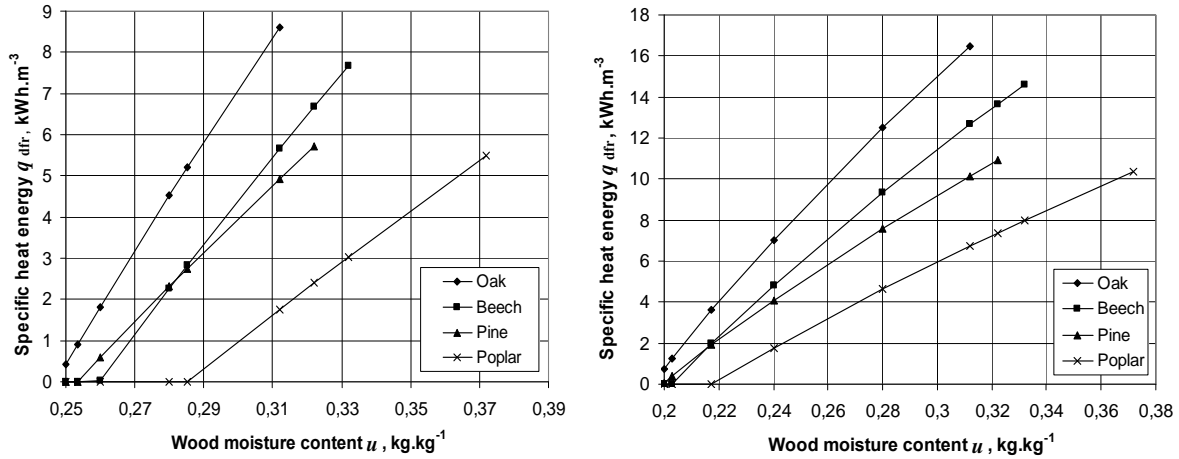


Figure 5: Change in  $q_{dfr}$  during defrosting of wood with  $t_{w0} = -10\text{ }^\circ\text{C}$  (left) and with  $t_{w0} = -20\text{ }^\circ\text{C}$  (right) depending on  $u$  and on the wood specie

The calculated according to equations (10), (1), and (8) change in  $q_{dfr}$  for frozen wood with  $u = 0.3\text{ kg}\cdot\text{kg}^{-1}$  and the calculated according to equation (5) change

in the content of non-frozen water in the wood  $u_{nfw}$  depending on initial wood temperature  $t_{w0}$  is shown on Fig. 6.

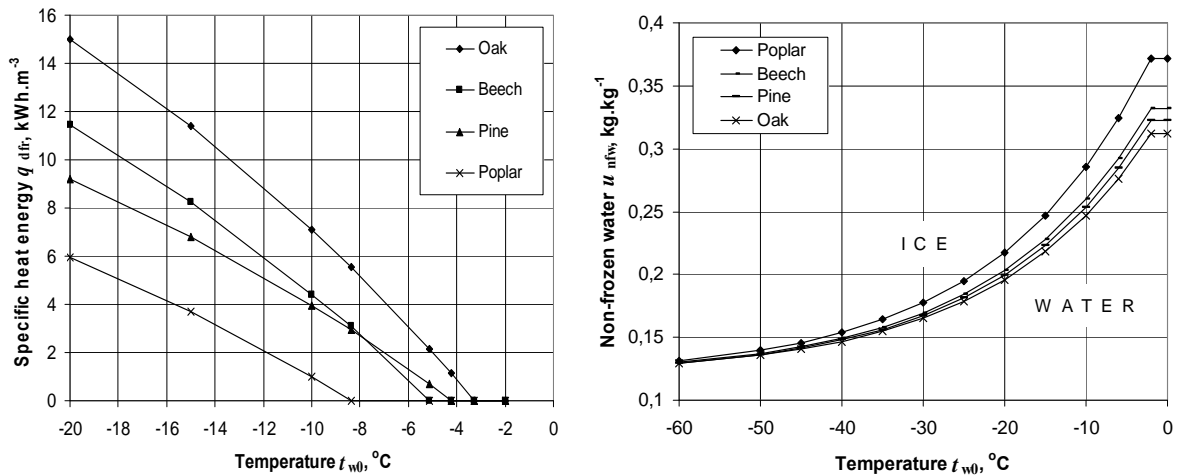


Figure 6: Change in  $q_{dfr}$  of wood with  $u = 0.3\text{ kg}\cdot\text{kg}^{-1}$  (left) and in  $u_{nfw}$  (right) during defrosting of wood materials depending on wood initial temperature  $t_{w0}$  and on the wood specie

#### 4. DISCUSSION

The analysis of the shown on Fig. 1 to 6 results gives the basis for the following conclusions:

1. The density of frozen wood  $\rho_w$  in the hygroscopic range increases according to a linear dependence when the wood moisture content  $u$  increases (Fig. 3 – left). A reason for

this is the proportionality of the wood mass depending on  $u$ .

2. The specific heat capacity of the frozen wood  $c_{wfr}$  and the specific heat capacity of the ice formed in the wood from the freezing of the bound water in it,  $c_{bw}$ , increase in the hygroscopic range according to a logarithmic dependences when the

wood moisture content  $u$  increases (Fig. 1 – right and Fig. 3 – right).

3. The specific heat energy consumption,  $q_{wfr}$ , which is needed for heating of frozen wood until melting of the ice in it in the hygroscopic range, increases almost proportionally to the wood moisture content  $u$  (Fig. 2).
4. The specific heat energy consumption,  $q_{bw}$ , which is needed for melting of the formed in the wood ice from the bound water in it, increases proportionally to the wood moisture content  $u$  (Fig. 4).
5. The specific energy, which is needed for defrosting of the wood in the hygroscopic range,  $q_{dfr}$ , increases almost proportionally to the wood moisture content  $u$  (Fig. 5).
6. The specific heat energy consumption  $q_{dfr}$  decreases according to a slight curvilinear dependence when the initial wood temperature  $t_{w0}$  increases (Fig. 6 – left).
7. The content of non-frozen water in the wood increases according to exponential dependence when the initial wood temperature  $t_{w0}$  increases (Fig. 6 – right).
8. When the wood contains the maximum possible quantity of bound water, i.e. when  $u = u_{fsp}^{271.15}$  (Chudinov 1968), for the defrosting of wood, which contains ice formed from the freezing of the mentioned water the following values for  $q_{dfr}$  are needed (Fig. 5):
  - $\text{kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and  $16.477\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$

for oak wood with  $u_{fsp}^{271.15} = 0.312\text{ kg}\cdot\text{kg}^{-1}$ ;

- $7.678\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and  $14.624\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for beech wood with  $u_{fsp}^{271.15} = 0.332\text{ kg}\cdot\text{kg}^{-1}$ ;
  - $5.711\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and  $10.903\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for pine wood with  $u_{fsp}^{271.15} = 0.372\text{ kg}\cdot\text{kg}^{-1}$ ;
  - $5.479\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and  $10.356\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for poplar wood with  $u_{fsp}^{271.15} = 0.322\text{ kg}\cdot\text{kg}^{-1}$ .
9. If the increase in  $q_{dfr}$  depending on  $u$  is taken to be fully linear in the range from  $u_{nfw}$  to  $u_{fsp}^{271.15}$  at which the thawing of the ice formed from the bound water in the wood is completed, then each increase of  $u$  by  $0.01\text{ kg}\cdot\text{kg}^{-1}$  in this range causes an increase in  $q_{dfr}$  for the studied wood species as follows:
    - by  $1.3260\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and by  $1.4168\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for oak;
    - by  $1.0634\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and by  $1.1328\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for beech;
    - by  $0.8325\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and by  $0.8886\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for pine;
    - by  $0.6312\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -10\text{ }^{\circ}\text{C}$  and by  $0.6694\text{ kWh}\cdot\text{m}^{-3}$  at  $t_{w0} = -20\text{ }^{\circ}\text{C}$  for poplar.

This results show that the most influencing factor on  $q_{dfr}$  is the basic density of the wood.

The obtained above results for the increase of  $q_{dfr}$  when  $u$  increases by

0.01 kg·kg<sup>-1</sup> at  $t_{w0} = -10$  °C differs by not more than 6.8% from the results for the increase of  $q_{wfr}$  at  $t_{w0} = -20$  °C.

Using the multiplied by 100 average values of the obtained above results for the increase of  $q_{bw}$  for the case of increase of  $u$

$$q_{dfr} = K_{dfr}(u - u_{nfw}) @ u_{nfw} < u \leq u_{fsp}^{271.15} \ \& \ T_{w0} < T_{dfr}, \quad (11)$$

where  $u$  is the wood moisture content in the hygroscopic range, kg·kg<sup>-1</sup>;

$K_{dfr}$  – coefficient equals to 137.14 for oak, 109.81 for beech, 86.06 for pine, and 65.03 for poplar wood;

$u_{nfw}$  – the content of non-frozen water in the wood at given temperature  $T_{w0} \leq 271.15$  K, kg·kg<sup>-1</sup>. The values of  $u_{nfw}$  can be easily determined with the help of Fig. 6 (right), which has been drawn using results obtained according to equation (5).

The calculated according to equation (11) values for  $q_{dfr}$  differ from their corresponding values of  $q_{dfr}$  on Fig. 5 and Fig. 6 (left) by not more than  $\pm 5\%$ . This accuracy of equation (11) is enough for different technological and/or engineering calculations of  $q_{dfr}$  in the whole hygroscopic range when  $u_{nfw} < u \leq u_{fsp}^{271.15} \ \& \ T_{w0} < T_{dfr}$  during wood defrosting.

Equation (11) can be used for calculation of  $q_{dfr}$  during wood defrosting for all wood species. Data for basic density,  $\rho_b$ , fiber saturation point,  $u_{fsp}^{293.15}$ , and volume shrinkage of the wood,  $S_v$ , for separate wood species needed for the determination of participating in this equation values of  $u_{nfw}$  and  $K_{dfr}$  can be found in the specialized literature (Shubin

by 0.01 kg·kg<sup>-1</sup> at  $t_{w0} = -10$  °C and  $t_{w0} = -20$  °C as proportionality coefficient,  $K_{dfr}$ , the value of  $q_{dfr}$  can be calculated according to the following equation:

1990, Pervan 2000, Trebula and Klement 2002, Videlov 2003, etc.).

## CONCLUSIONS

In the present paper easy for engineering applications mathematical model of the specific heat energy consumption needed for defrosting of frozen wood materials in the hygroscopic range,  $q_{dfr}$ , has been suggested. The model takes into account the influence on  $q_{dfr}$  of the following factors: wood moisture content, initial wood temperature of the frozen wood, basic density of the wood, volume shrinkage of the wood, and for the first time the fiber saturation point  $u_{fsp}$  of separate wood species and the influence of the temperature on  $u_{fsp}$ .

For the calculation of  $q_{dfr}$  according to the suggested model a software program has been prepared in the calculation environment of Visual Fortran Professional. Using the program computations have been carried out for the determination  $q_{dfr}$  of oak, pine, beech, and poplar frozen wood with initial temperatures in the range from  $t = -20$  °C to  $t = -2$  °C, at which the thawing of the ice formed from the bound water is completed (Chudinov 1968) and with moisture content in the hygroscopic range  $0.2 \text{ kg} \cdot \text{kg}^{-1} \leq u \leq u_{fsp}$  during wood defrosting.

Based on the obtained results, a very simple and easy for use equation for the calculation of  $q_{\text{dff}}$  depending only on the wood moisture content and on the content of non-frozen water in the wood at given initial wood temperature has been suggested. This equation can be used for precise enough ( $\pm 5\%$ ) technological and engineering calculations of various processes for thermal and hydro-thermal treatment of frozen wood materials aimed at their defrosting.

The obtained results are of specific importance for the optimization of the technology and for the model based automatic control (Deliiski 2003, 2004) of different defrosting and/or frosting processes of wood and other capillary porous materials.

The suggested model and the obtained results can be of interest also for the educational process at specialized technical universities and engineering schools.

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