

## DEFORMATION OF SHAFTS AND AXLES LOADED WITH TRANSVERSE FORCE APPLIED IN THEIR CONSOLE PARTS

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### ABSTRACT

The methodology for calculation of the shafts and the axles subjected to deformation taking into account the radial clearance in the bearings is presented in the paper.

Deflection of the shafts and axles is determined for two cases: the first one is when the antifriction bearings are assumed to be perfectly solid supports and the second one is when the above-mentioned are assumed to be elastic.

The deflection of the antifriction bearings is a sum of the initial clearance and the elastic deformations of the antifriction bodies and their rings.

The aim of the present work is to determine the elastic deformations of the antifriction bodies and the rings of the bearings.

**Key words:** shafts, axles, antifriction bearings, deflection, radial clearance

### INTRODUCTION

Deformations (elastic deflections) of the shafts and the axles cause a harmful impact on the joints related to them (key-joints, splines, etc.), as well as on the bearings, gear-drives and other details and junctions of the machines. The aforesaid increase the stress concentration, decrease the fatigue resistance of the parts, accelerate the wear-out, and worsen the work precision of the mechanisms and machines. Considerable deflections of the shafts and axles could bring to wedging of the bearings, to aggravation of the frequency characteristics of the machines, to formation of vibrations, etc. [S. Sokolovski, N. Deliiski 2010; N. Staneva, V. Vlasev, 2013].

### 1. DIFFERENT TYPE OF DEFLECTIONS OF THE SHAFTS AND AXLES

Bending of the shafts and axles in their console part where the operating bodies of the wood-working machines are situated (circular saws, band-saw belts, etc.), has a

negative influence on their work. The resistance of the shafts against vibrations and the rotation precision are decreased.

The unfavorable influence of the deflections of the shafts and axles requires their exact determination and seeking for their diminution. On fig. 1 is shown deflection of shafts and axles loaded with a force exercising an effort outside the supports which is the most common case about wood-working machines. In the cases when the shafts are loaded on both their endings with forces (band-saws, circular blades, etc.) the deflections are determined on both endings of the shaft, in the sections where the external forces are applied.

The total load of the shafts and axles  $f$  in their console part is shown on fig. 1B as a sum of the deflection  $f_1$  in perfectly solid supports (fig.1a) and the deflection  $f_2$  in elastic supports, i.e.

$$f = f_1 + f_2. \quad (1)$$

On fig. 1a are shown the deflection  $f_1$  and the type of the elastic line of the shaft or the axle after their bending at perfectly solid

supports. Generally, in these cases, the antifriction bearings are admitted to be perfectly solid supports.

The deflection  $f_1$  is defined by the relation [S. Sokolovski, 2007]:

$$f_1 = \frac{Fa^2}{3E \cdot I}(a+b), \quad (2)$$

Where  $f_1$  is the deflection of the shaft or the axle under the action of the external force (in the console part), m;

$F$  – external force of the shaft or the axle, N;

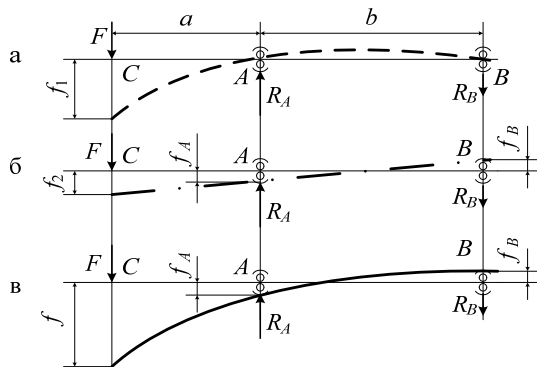
$I$  – moment of inertia of the section of the equivalent shaft or axle with a constant diameter,  $m^4$ . It is determined by the well-known formula

$$I = \frac{\pi d_{ekv}^4}{64}; \quad (3)$$

$d_{ekv}$  – diameter of the equivalent shaft or axle with a constant diameter, m;

$E$  – modulus of linear deformation of the shaft or axle material, Pa;

$a, b$  – distances according to fig. 1, m.



**Figure 1: Shafts and axles deflection loaded with a force outside the supports: a – at perfectly solid supports; б – at elastic supports; B – total deflection**

In fact the supports with antifriction bearings are elastic. The bigger the elastic yield (displacement) of one bearing subjected to a single load is, the smaller its rigidity is. When putting bearings on mandrels and operating shafts of woodworking machines the rigidity of the

bearing is of a considerable significance as the yield (displacement) in the bearing changes the type of the elastic line, the theoretically calculated deflections and thence – the accuracy of the elaborated detail.

The rigidity of the bearing depends on its construction, on the form of the rolling bodies etc. In this report the factors on which depend the displacements of the bearing are examined. On fig. 1б is shown the deflection  $f_2$  of perfectly rigid shafts and axles depending on the displacements in the bearings  $f_A$  and  $f_B$ . This deflection is determined geometrically by the similarity of the two right triangles  $\Delta BAA_1$  and  $\Delta BCC_1$  (fig. 2), for which could be written the following equality

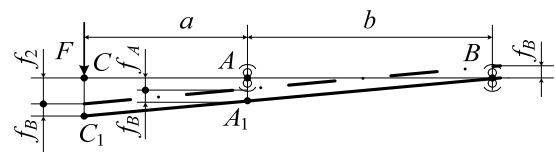
$$\frac{b}{a+b} = \frac{f_A + f_B}{f_2 + f_B}, \quad (4)$$

Whence it follows that

$$f_2 = \frac{a}{b}(f_A + f_B) + f_A, \quad (5)$$

Where  $f_2$  is the deflection of the perfectly rigid shafts and axles under the action of the external force depending on the displacements of the bearings, m;

$f_A, f_B$  – displacements of the bearings in the supports, m.



**Figure 2: Scheme of geometrical determination of the deflection of shafts and axles in accordance with the displacements in the shafts**

The displacement of the shafts is a sum of their initial clearance  $G_r$  and the elastic deformations of the rolling bodies and rings  $\delta$ :

$$f_A = G_{rA} + \delta_A - \text{for the bearing in support A, } (6)$$

$f_B = G_{rB} + \delta_B$  - for the bearing in support  $B$ , (7)

where  $G_{rA}$  and  $G_{rB}$  are the initial radial clearances respectively for the bearing in support  $A$  and for the bearing in support  $B$ . Their values could be found in the catalogues for rolling bearings;

$\delta_A, \delta_B$  – elastic deformations of the most loaded rolling bodies and rings of the bearings, respectively for the bearing in support  $A$  and for the bearing in support  $B$ .

## 2. DETERMINATION OF DISPLACEMENT IN BEARINGS, PROVOKED BY THE ELASTIC DEFORMATIONS OF THE ROLLING BODIES AND RINGS

It is known that in the case of the radial bearings, the rolling bodies which are disposed in the loaded zone of the rings, they receive different parts of the total force. The most loaded is the lowest rolling body which is situated in the plane of action of the radial force  $R$ . The load could be defined by taking into consideration the clearance in the bearing according to the following relation [K. Kolesnikov and others, 2004]:

$$Q = \frac{5R}{z}, \quad (8)$$

where  $Q$  is the force acting on the lowest rolling body of the bearing, N;

$R$  – the radial load of the bearing (support reactions of the shaft), N; (respectively  $R_A$  - for the bearing in support  $A$ ;  $R_B$  – for the bearing in support  $B$ );

$z$  – the number of the bearing balls which is taken by the catalogues for antifriction bearings.

Under the action of the force  $Q$  in the zones of contact of the rolling body with the rings (fig. 3a) occur elastic contact deformations of the rolling body and the rings shown on the scheme on fig. 3б. The rolling body changes its form from a circle to an ellipse, and hollows (small pits).

For the determination of the joint displacement of the most loaded rolling body and the rings of the bearing, let us consider their elastic deformations between the internal ring and the rolling body (point 1 on fig. 3a) and the deformations between the external ring and the rolling body (point 2 on fig. 3a). After the elastic deformations occur the center of the bearing, consequently the center of the shaft will be translocated on a distance  $\delta$  which is the sum of the deformations of the contact in point 1 -  $\delta_1$  and the contact in point 2 -  $\delta_2$ , i.e.

$$\delta = \delta_1 + \delta_2. \quad (9)$$

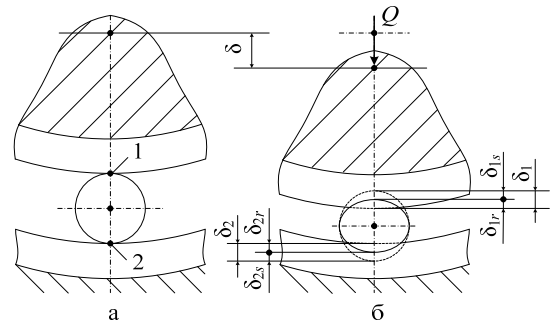


Figure 3: Scheme of the deformations in an antifriction bearing: a – before loading; б – after loading

The shift in the contact of the point 1 (fig. 3a) between the internal ring and the rolling body  $\delta_1$  is a sum of the deformations of the rolling body (a ball or a roller)  $\delta_{1s}$  and of the internal ring  $\delta_{1r}$ , i.e.

$$\delta_1 = \delta_{1s} + \delta_{1r}. \quad (10)$$

The shift in the contact of the point 2 (fig. 3a) between the external ring and the rolling body  $\delta_2$  is a sum of the deformations of the rolling body (a ball or a roller)  $\delta_{2s}$  and of the external ring  $\delta_{2r}$ , i.e.

$$\delta_2 = \delta_{2s} + \delta_{2r}. \quad (11)$$

To determine the deformations  $\delta_1$  of the contact in point 1 (fig. 3a) we could use the relations at the contact of a sphere or a cylinder with convex surfaces, whereas for the deformations  $\delta_2$  of the contact in point 2

(fig. 3a) we can use the relations at the contact of a sphere or a cylinder with concave surfaces. When specifying these deformations (displacements) it is necessary to be expressed in dependence with the applied force in the bearing. The applied force of the bearing is perceived by a very small surface of contact of the rolling bodies and the rings and therefore the strains in these contacts are considerable which provokes elastic deformations.

On the basis of the theory of the German physicist Herz [H. Herz, 1881] are obtained formulae for determination of the contact surface (which has the form of an ellipse in the ball bearings and a rectangle in the roller bearings), the normal strains and deformations between sphere and plane,

sphere and sphere, cylinder and plane and cylinder and cylinder.

The deformation  $\delta$  in ball bearings (point contact) of the most loaded ball with a force  $Q$  is equal to [L. Perel, 1982]

$$\delta = 2.79 \cdot 10^{-6} K (Q^2 \cdot \rho_{\Sigma})^{\frac{1}{3}}, \quad (12)$$

where  $\delta$  is the deformation of the most loaded ball in the case of ball-bearings, m;

$Q$  – force exercising an effort on the lowest rolling body of the bearing, N.

It is determined with the help of formula (8);

$K$  – parameter characterizing the curve of the contacting surface. It is determined by table 1 [L. Perel, 1982] in accordance to the subsidiary parameter  $\tau$ , characterizing the difference of curviness of the contact surface.

**Table 1: Values of the subsidiary parameter  $\tau$  and the parameter  $K$**

$\tau$	$K$	$\tau$	$K$	$\tau$	$K$	$\tau$	$K$	$\tau$	$K$	$\tau$	$K$
0,999	0,207	0,989	0,393	0,979	0,465	0,968	0,518	0,946	0,588	0,90	0,680
0,998	0,249	0,988	0,402	0,978	0,470	0,966	0,526	0,942	0,598	0,88	0,709
0,997	0,279	0,987	0,410	0,977	0,476	0,964	0,533	0,938	0,608	0,84	0,755
0,996	0,302	0,986	0,420	0,976	0,481	0,962	0,540	0,934	0,618	0,80	0,792
0,995	0,320	0,985	0,427	0,975	0,486	0,960	0,546	0,930	0,626	0,70	0,859
0,994	0,336	0,984	0,433	0,974	0,491	0,958	0,553	0,926	0,634	0,60	0,904
0,993	0,350	0,983	0,440	0,973	0,495	0,956	0,559	0,922	0,642	0,50	0,938
0,992	0,362	0,982	0,447	0,972	0,500	0,954	0,565	0,918	0,615	0,40	0,962
0,991	0,373	0,981	0,453	0,971	0,505	0,952	0,571	0,914	0,657	0,20	0,991
0,990	0,384	0,980	0,459	0,970	0,509	0,950	0,577	0,910	0,664	0	1
<b>Note. Intermediate values are obtained through interpolation</b>											

The subsidiary parameter  $\tau$  is determined in dependence with the type of contact (sphere or cylinder) and of the form of the contact surfaces (convex or concave) [L. Perel, 1982]:

- for radial ball single-row and radial ball spherical bearings:

in contact of a ball with the internal ring  $\tau_1$

$$\tau_1 = \frac{1.9(D_0 - D_w \cos \alpha) + 2D_w \cos \alpha}{2.1(D_0 - D_w \cos \alpha) + 2D_w \cos \alpha}; \quad (13)$$

in contact of the ball with the external ring  $\tau_2$

$$\tau_2 = \frac{1.9(D_0 - D_w \cos \alpha) - 2D_w \cos \alpha}{2.1(D_0 - D_w \cos \alpha) - 2D_w \cos \alpha}; \quad (14)$$

- for radial roller spherical bearings with barrel-shaped rollers:

in contact of a barrel-shaped roller with the internal ring  $\tau_1$

$$\tau_1 = \frac{\frac{2D_0}{D_0 - D_w \cos \alpha} - 3 \cdot 10^{-4} D_w}{\frac{2D_0}{D_0 - D_w \cos \alpha} + 3 \cdot 10^{-4} D_w}; \quad (15)$$

in contact of a barrel-shaped roller with the external ring  $\tau_2$

$$\tau_2 = \frac{\frac{2D_0}{D_0 + D_w \cos \alpha} - 3 \cdot 10^{-4} D_w}{\frac{2D_0}{D_0 + D_w \cos \alpha} + 3 \cdot 10^{-4} D_w}. \quad (16)$$

In the formulas (13) – (16)  $D_w$  is the diameter of the rolling bodies (balls or rollers), m;  $\alpha$  is the angle of contact, °;  $D_0$  is the diameter of the circumference on which the rolling bodies are disposed, m. It is determined as the average diameter of the internal  $d$  and external  $D$  diameters of the bearing, i.e.

$$D_0 = 0.5(d + D). \quad (17)$$

The values of  $D_w$ ,  $\alpha$ ,  $d$  and  $D$  are given in the catalogues for antifriction bearings.

$\rho_{\Sigma}$  - the sum of the curve,  $m^{-1}$ . It is determined in accordance to the type of the contact (sphere or cylinder) and to the form of the contact surfaces (convex or concave) [L. Perel, 1982]:

- for radial ball single-row and radial ball spherical bearings:

in contact of a ball with the internal ring  $\rho_{\Sigma 1}$

$$\rho_{\Sigma 1} = \frac{1}{D_w} \left( 2.1 + \frac{2D_w \cos \alpha}{D_0 - D_w \cos \alpha} \right); \quad (18)$$

in contact of a ball with the external ring  $\rho_{\Sigma 2}$

$$\rho_{\Sigma 2} = \frac{1}{D_w} \left( 2.1 - \frac{2D_w \cos \alpha}{D_0 + D_w \cos \alpha} \right); \quad (19)$$

- radial roller spherical bearings with barrel-shaped rollers:

in contact of a barrel-shaped roller with the internal ring  $\rho_{\Sigma 1}$

$$\rho_{\Sigma 1} = \frac{1}{D_w} \left( \frac{2D_0}{D_0 - D_w \cos \alpha} + 3 \cdot 10^{-4} D_w \right); \quad (20)$$

in contact of a barrel-shaped roller with the external ring  $\rho_{\Sigma 2}$

$$\rho_{\Sigma 2} = \frac{1}{D_w} \left( \frac{2D_0}{D_0 + D_w \cos \alpha} + 3 \cdot 10^{-4} D_w \right); \quad (21)$$

- radial roller bearings with cylindrical rollers:

in contact of the cylindrical roller with the internal ring  $\rho_{\Sigma 1}$

$$\rho_{\Sigma 1} = \frac{2D_0}{D_w(D_0 - D_w)}; \quad (22)$$

in contact of the cylindrical roller with the external ring  $\rho_{\Sigma 2}$

$$\rho_{\Sigma 2} = \frac{2D_0}{D_w(D_0 + D_w)}; \quad (23)$$

where  $D_w$  – the diameter of the rolling bodies (balls or rollers), m. The values can be found in the catalogues for antifriction bearings;

$D_0$  – diameter of the circumference on which the rolling bodies are disposed, m, determined with the help of formula (17);

$\alpha$  – contact angle, °. It is determined due to the catalogues for antifriction bearings.

Deformation  $\delta$  in roller bearings (linear contact) of the most loaded roller with force  $Q$  is [L. Perel, 1982]

$$\delta = 1.1 \cdot 10^{-10} \frac{Q^{0.925}}{l^{0.85}}, \quad (24)$$

where  $\delta$  is the deformation of the most loaded roller in roller bearings, m;

$Q$  – the force charging the lowest rolling body of the bearing, N. It is described with the help of formula (8);

$l$  – the length of the rollers, m, given in the catalogues for antifriction bearings.

Verification of the proposed methodology is made by the following example:

Determination of the total deflection of shaft (fig. 1), loaded in its console part with a force  $F = 1500$  N. Diameters of the shaft

are: in the console part, which is equal to the equivalent  $d_C = d_{ekv} = 0.03$  m, diameters of the bearing necks are  $d_A = d_B = 0.04$  m. The shaft is elaborated from quality steel 45. The shaft is mounted in bearings on two similar radial ball single-row bearings №6308 with  $d = 0.04$  m,  $D = 0.09$  m,  $D_0 = 0.065$  m,  $D_w = 0, 01508$  m,  $z = 8$  and  $\alpha = 0^\circ$ . The bearing body is shared by the two bearings which provide the coaxiality of the bearings. The distances are:  $a = 0.2$  m,  $b = 0.4$  m.

Support reactions are (fig. 1):

$$R_A = \frac{F(a+b)}{b} = \frac{1500(0.2+0.4)}{0.4} = 2250 \text{ N};$$

$$R_B = -\frac{Fa}{b} = \frac{1500 \cdot 0.2}{0.4} = 750 \text{ N}.$$

Total deflection of the shaft  $f$  is

$$f = f_1 + f_2 = 1428 \cdot 10^{-6} + 86 \cdot 10^{-6} = 1514 \cdot 10^{-6} \text{ m}.$$

Deflection  $f_1$  of the shaft under the action of the external force is

$$f_1 = \frac{Fa^2}{3E \cdot I} (a+b) = \frac{1500 \cdot 0.2^2 (0.2+0.4)}{3 \cdot 2.1 \cdot 10^{11} \cdot 0.004 \cdot 10^{-5}} = 1428 \cdot 10^{-6} \text{ m}.$$

The moment of inertia of the equivalent shaft section with constant diameter is

$$I = \frac{\pi d_{ekv}^4}{64} = \frac{3.14 \cdot 0.03^4}{64} = 0.004 \cdot 10^{-5} \text{ m}^4;$$

$E = 2.1 \cdot 10^{11}$  Pa – modulus of linear deformation of steel.

he deflection  $f_2$  of the shaft under the action of the external force depending on displacements in the bearings is

$$f_2 = \frac{a}{b}(f_A + f_B) + f_A = \frac{0.2}{0.4}(46 \cdot 10^{-6} + 34 \cdot 10^{-6}) + 46 \cdot 10^{-6} = 86 \cdot 10^{-6} \text{ m}.$$

The displacement of the bearings is:

- for the bearing in support A

$$f_A = G_{rA} + \delta_A = 23 \cdot 10^{-6} + 23 \cdot 10^{-6} = 46 \cdot 10^{-6} \text{ m}.$$

The initial clearance of the antifriction bearing is  $G_{rA} = 23 \cdot 10^{-6}$  m.

The elastic deformation  $\delta_A$  of the bearing in the support A is

$$\delta_A = \delta_{A1} + \delta_{A2} = 11.8 \cdot 10^{-6} + 11.2 \cdot 10^{-6} = 23 \cdot 10^{-6} \text{ m}.$$

The elastic deformations in the contact in point 1 (the ball with the internal ring) is

$$\delta_{A1} = 2.79 \cdot 10^{-6} K_{A1} (Q_A^2 \cdot \rho_{\Sigma A1})^{\frac{1}{3}} = 2.79 \cdot 10^{-6} \cdot 0.603 (1406^2 \cdot 0.000178)^{\frac{1}{3}} = 11.8 \cdot 10^{-6} \text{ m}.$$

The force exercising an effort on the lowest ball

$$Q_A = \frac{5R_A}{z} = \frac{5 \cdot 2250}{8} = 1406 \text{ N}.$$

Parameter  $K_{A1}$  from table 1 depending on the subsidiary parameter  $\tau_{A1}$

$$\tau_{A1} = \frac{1.9(D_0 - D_w \cos \alpha) + 2D_w \cos \alpha}{2.1(D_0 - D_w \cos \alpha) + 2D_w \cos \alpha} = \frac{1.9(0.065 - 0.01508 \cdot 1) + 2 \cdot 0.01508 \cdot 1}{2.1((0.065 - 0.01508 \cdot 1) + 2 \cdot 0.01508 \cdot 1)} = 0.940.$$

$$3a \tau_{A1} = 0.940; K_{A1} = 0.603.$$

The sum of the curve is

$$\rho_{\Sigma A1} = \frac{1}{D_w} \left( 2.1 + \frac{2D_w \cos \alpha}{D_0 - D_w \cos \alpha} \right) = \frac{1}{0.01508} \left( 2.1 + \frac{2 \cdot 0.01508}{0.065 - 0.01508} \right) = 0.000178 \text{ m}^{-1}.$$

The elastic deformations in the contact of point 2 (the ball with the external ring)

$$\delta_{A2} = 2.79 \cdot 10^{-6} K_{A2} (Q_A^2 \cdot \rho_{\Sigma A2})^{\frac{1}{3}} = 2.79 \cdot 10^{-6} \cdot 0.695 (1406^2 \cdot 0.0001)^{\frac{1}{3}} = 11.2 \cdot 10^{-6} \text{ m}.$$

Parameter  $K_{A2}$  from table 1 depending on the subsidiary parameter  $\tau_{A2}$

$$\tau_{A21} = \frac{1.9(D_0 - D_w \cos \alpha) - 2D_w \cos \alpha}{2.1(D_0 - D_w \cos \alpha) - 2D_w \cos \alpha} = \frac{1.9(0.065 - 0.01508 \cdot 1 - 2 \cdot 0.01508 \cdot 1)}{2.1((0.065 - 0.01508 \cdot 1 - 2 \cdot 0.01508 \cdot 1))} = 0.89.$$

$$3a \tau_{A2} = 0.89; K_{A2} = 0.695.$$

The sum of the curve is

$$\rho_{\Sigma A2} = \frac{1}{D_w} \left( 2.1 - \frac{2D_w \cos \alpha}{D_0 - D_w \cos \alpha} \right) = \frac{1}{0.01508} \left( 2.1 - \frac{2 \cdot 0.01508}{0.065 - 0.01508} \right) = 0.0001 \text{ m}^{-1}.$$

- for the bearing in support B

$$f_B = G_{rB} + \delta_B = 23 \cdot 10^{-6} + 11 \cdot 10^{-6} = 34 \cdot 10^{-6} \text{ m}.$$

The initial clearance of the antifriction bearing is  $G_{rB} = 23 \cdot 10^{-6} \text{ m}$ .

The elastic deformations  $\delta_B$  for the bearing in the support B is

$$\delta_B = \delta_{B1} + \delta_{B2} = 5.6 \cdot 10^{-6} + 5.4 \cdot 10^{-6} = 11 \cdot 10^{-6} \text{ m}.$$

The elastic deformations in the contact of point 1 (the ball with the internal ring)

$$\delta_{B1} = 2.79 \cdot 10^{-6} K_{B1} (Q_B^2 \cdot \rho_{\Sigma B1})^{\frac{1}{3}} = 2.79 \cdot 10^{-6} \cdot 0.603 (468.8^2 \cdot 0.000178)^{\frac{1}{3}} = 5.6 \cdot 10^{-6} \text{ m}.$$

The force exercising an effort on the lowest ball

$$Q_B = \frac{5R_B}{z} = \frac{5 \cdot 750}{8} = 468.8 \text{ N}.$$

Parameter  $K_{B1}$  from table 1 depending on the subsidiary parameter  $\tau_{B1}$

$$\tau_{B1} = \frac{1.9(D_0 - D_w \cos \alpha) + 2D_w \cos \alpha}{2.1(D_0 - D_w \cos \alpha) + 2D_w \cos \alpha} = \frac{1.9(0.065 - 0.01508 \cdot 1 + 2 \cdot 0.01508 \cdot 1)}{2.1((0.065 - 0.01508 \cdot 1 + 2 \cdot 0.01508 \cdot 1))} = 0.940.$$

$$3a \tau_{B1} = 0.940; K_{B1} = 0.603.$$

The sum of the curve is

$$\rho_{\Sigma B1} = \frac{1}{D_w} \left( 2.1 + \frac{2D_w \cos \alpha}{D_0 - D_w \cos \alpha} \right) = \frac{1}{0.01508} \left( 2.1 + \frac{2 \cdot 0.01508}{0.065 - 0.01508} \right) = 0.000178 \text{ m}^{-1}.$$

The elastic deformations in the contact of point 2 (the ball with the external ring)

$$\delta_{B2} = 2.79 \cdot 10^{-6} K_{B2} (Q_A^2 \cdot \rho_{\Sigma B2})^{\frac{1}{3}} = 2.79 \cdot 10^{-6} \cdot 0.695 (468.8^2 \cdot 0.0001)^{\frac{1}{3}} = 5.4 \cdot 10^{-6} \text{ m}.$$

Parameter  $K_{B2}$  from table 1 depending on the subsidiary parameter  $\tau_{B2}$

$$\tau_{B2} = \frac{1.9(D_0 - D_w \cos \alpha) - 2D_w \cos \alpha}{2.1(D_0 - D_w \cos \alpha) - 2D_w \cos \alpha} = \frac{1.9(0.065 - 0.01508 \cdot 1 - 2 \cdot 0.01508 \cdot 1)}{2.1((0.065 - 0.01508 \cdot 1 - 2 \cdot 0.01508 \cdot 1))} = 0.89.$$

$$3a \tau_{B2} = 0.89; K_{B2} = 0.695.$$

The sum of the curve is

$$\rho_{\Sigma B2} = \frac{1}{D_w} \left( 2.1 - \frac{2D_w \cos \alpha}{D_0 - D_w \cos \alpha} \right) = \frac{1}{0.01508} \left( 2.1 - \frac{2 \cdot 0.01508}{0.065 - 0.01508} \right) = 0.0001 \text{ m}^{-1}.$$

## CONCLUSION

The results of the example show that the most important deflection of the shafts and the axles is in the console part from the action of the external load. It could be reduced by decreasing the console part, if it is possible, or by increasing the diameter of the shaft. The deflection of shafts and axles caused by the deformations in the bearings is considerably smaller than the total deflection (for the case considered above it is about 6 %). The deflection of shafts and axles caused by the deformations in the bearings is necessary to be done when mounting bearings of precise metal and woodworking machines, centrifuges,

ventilators, electro motors and etc. It is imperative to determine the deflection of the shafts and axles caused by the deformations in the bearings when mounted on bearings of exact metal and woodworking machines, centrifuges, ventilators, electro motors and etc. In conventional machines it is advisable.

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