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INVESTIGATIONS OF THE FORCED TORSIONAL VIBRATIONS IN THE SAW UNIT OF A KIND OF WOOD SHAPERS, USED IN THE WOOD PRODUCTION

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ABSTRACT

The proposed study presents a numerical investigation of the forced torsional vibrations in the wood shaper's saw unit caused by the variable moments on the drive electric motor and the wood shaper's saw. The influence on the vibrations of the blades' number of the saw, which really work, is investigated. This research is done on the base of a concrete mechanic-mathematical model for investigation of the torsional vibrations of a wood shaper's saw developed by the authors. The main features in the construction of this kind of wood shapers are rendered an account in this model. The investigation`s results can be used as a base for making some concrete and well-founded recommendations concerning the operation of these machines. These recommendations are important for increase of reliability of the wood shapers as well as the accuracy and quality of their production.

Key words: wood shapers, modeling, torsional vibrations

INTRODUCTION

Discovering and investigation of the causes for originate of the intensive torsional vibration of the wood shapers requires understanding the essence of the dynamic processes in them, when machine works. It is necessary to conduct purposeful studies in which the machine can be considered as a mechanical vibrating system with known characteristics of its individual elements (Amirouche 2006). For this purpose, firstly it is necessary to have mechanical-mathematical modeling and composing of equations describing the vibration of the elements of the wood shaper. Well-targeted research can be done by solving these equations in different conditions. Some recommendations for the construction's design and the work regimes of the machine are formed on their base (Bachev *at al*., 2012).

The kind of wood shapers that are commonly used in the practice of forestry industry (Filipov 1977) are examined in the proposed study. Fig. 1 shows the general view, and Fig. 2 – a scheme of this type of wood shapers (Obreshkov 1997). The machine body is marked with *1*, *2* is the electric motor, *3* – the belt drive, which can be a wedge belt or a ribbed belt, *4* – the spindle with the bearings, *5* – the arbor with morse cone, *6* – the work table, *7* – wood shaper's saw.

Figure 1: Wood shaper general view

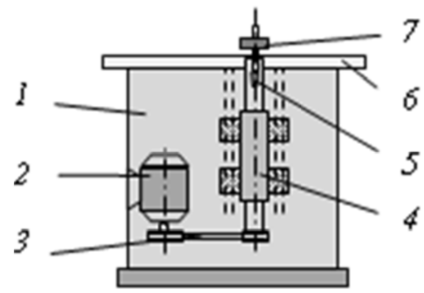


Figure 2: Scheme of the wood shaper

Some special features in the modeling of woodworking machines are examined in previous works of the authors (Vukov *at al.*, 2010). Some numerical investigations of the natural frequencies and mode shapes of the wood shapers are made. Some based recommendations for the avoidance of resonant regimes are formed on the basis of this study. Practically it is connected with the increase of reliability of the machine as well as with the accuracy and quality of the production.

The goal of this study is to make a numerical investigation of the forced torsional vibrations in the wood shaper's saw unit due to the variable moments on the drive electric motor and on the wood shaper's saw. The main aim is to study the influence on the torsional vibrations of the blades' number of the saw, which really work. The investigation is done on the base of an adequate mechanic-mathematical model for investigation of the torsional vibrations of the wood shapers developed by the authors. The model presents features in the construction of a kind of wood shapers.

MECHANIC-MATHEMATICAL MODEL

A mechanic-mathematical model for investigation of the dynamical processes and vibrations in the wood shaper's saw unit is built by the authors. The model is shown on the fig. 3. This model includes four discrete mass connected with three massless elastic elements. φ_i, $i = 1, 2, 3, 4$ are the angles of the rotation of the corresponding rotor. The elasticity coefficients of the electric motor's shaft, the belt and the spindle are taken into account. The elasticity angular coefficient of the electric motor's shaft is marked with c_1, and this one of the spindle – with c_3 (*N.m/rad*). The elasticity linear coefficients of the two parts of the belt between the belt puller are c_{23} and c_{32} (*N/m*). The damping coefficients are marked with b and respective indices. The applied moments M_i on the disks are shown too.

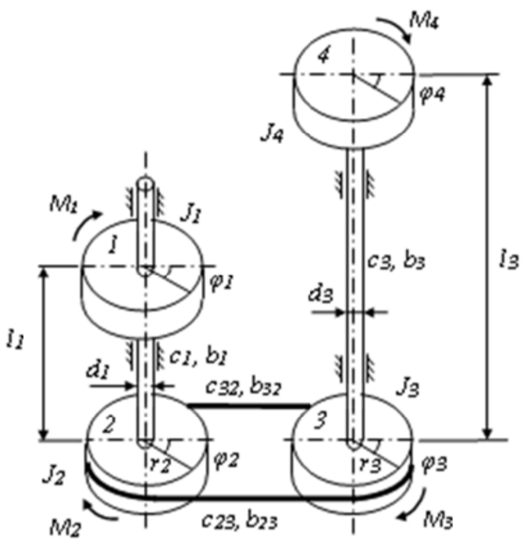


Figure 3: Mechanic-mathematical model

The necessary reduced mass inertia moments ($kg{\cdot}m^2$) are rendered in account (fig. 3): They are J_1 – the mass inertia moment of the electric motor's rotor; J_2 –

the mass inertia moment of the belt puller on the electric motor's shaft; J_3 – the mass inertia moment of the belt puller on the spindle; J_4 – the mass inertia moment of the wood shaper's saw with cutter arbor.

Some other symbols on fig. 3 are: d_1, d_3 – diameters of the electric motor's shaft and spindle (m); l_1, l_3 – computing length of the electric motor's shaft and spindle (m); r_2, r_3 – radius of the belt pullers on the electric motor's shaft and spindle (m); G – modulus of shearing.

The investigation of the torsional vibrations of the wood shaper's saw unit requires formulation and solution of the differential equations which describe these processes. Therefore, the priority of the matrix mechanics is used (Angelov and Slavov 2010).

The mechanic-mathematical model is done by using the applied engineer program (Mathematica). An algorithm for formulation of the matrixes which describe the properties of the mechanical system is developed. The differential equations which describe the vibrations are deduced by using the Lagrange's method.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right) - \frac{\partial E_K}{\partial q} + \frac{\partial E_P}{\partial q} = Q - \frac{\partial F}{\partial \dot{q}}, \qquad (1)$$

where q_i are the generalized coordinates, E_K and E_P are respectively the kinetic and the potential energy of the multibody systems, F is dissipative function, Q is the vector of the generalized forces.

The vector of the generalized coordinates is $\mathbf{q} = [\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4]^T$.

The kinetic energy of the mechanical system is obtained as a sum of the kinetic energy of the four basic bodies (the electric motor's rotor, the belt puller on the electric motor's shaft, the belt puller on the spindle, wood shaper's saw). By analogy the potential energy of the mechanical system is obtained as a sum of the potential energies received from the deformations of the electric motor's shaft, the belt and the spindle

$$E_K = \frac{1}{2}J_1.\dot{\varphi}_1^{\,2} + \frac{1}{2}J_2.\dot{\varphi}_2^{\,2} + \frac{1}{2}J_3.\dot{\varphi}_3^{\,2} + \frac{1}{2}J_4.\dot{\varphi}_4^{\,2},$$

$$\begin{aligned} E_P = {} & \frac{1}{2}c_1.(\varphi_1 - \varphi_2)^2 + \frac{1}{2}c_{23}.(r_2.\varphi_2 - r_3.\varphi_3)^2 + \\ & + \frac{1}{2}c_{32}.(r_3.\varphi_3 - r_2.\varphi_2)^2 + \frac{1}{2}c_3.(\varphi_3 - \varphi_4)^2 \end{aligned}. \qquad (2)$$

The dissipative function is

$$\begin{aligned} F = {} & \frac{1}{2}b_1.(\dot{\varphi}_1 - \dot{\varphi}_2)^2 + \frac{1}{2}b_{23}.(r_2.\dot{\varphi}_2 - r_3.\dot{\varphi}_3)^2 + \\ & + \frac{1}{2}b_{32}.(r_3.\dot{\varphi}_3 - r_2.\dot{\varphi}_2)^2 + \frac{1}{2}b_3.(\dot{\varphi}_3 - \dot{\varphi}_4)^2 \end{aligned}. \qquad (3)$$

The vector of the generalized loads includes all torsional moments, applied on the rotors. It is

$$\mathbf{Q} = [M_1 \quad -M_2 \quad -M_3 \quad -M_4]^T, \qquad (4)$$

where: M_1 is the moment of the electric motor; M_2 and M_3 – moments of the belt pullers from their interaction with the belt; M_4 – the moment of the wood shaper's saw, which is formed when the machine works.

This method supposes receiving a system of parametric linear differential equations which describe the small forced torsional vibrations of the saw unit. They are

$$\mathbf{M}.\ddot{\mathbf{q}} + \mathbf{B}.\dot{\mathbf{q}} + \mathbf{C}.\mathbf{q} = \mathbf{Q} \qquad (5)$$

The matrix, which characterize the mass-inertial properties of the mechanical system (numerical values are given in $kg.m^2$), the elastic properties ($\frac{N.m}{rad}$) and the damping properties ($\frac{N.m.s}{rad}$), are

$$\mathbf{M} = \left[a_{ij}\right], \qquad a_{ij} = \frac{\partial^2 E_K}{\partial \dot{q}_i . \partial \dot{q}_j},$$

$$\mathbf{M} = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix} = \begin{bmatrix} 0{,}0102 & 0 & 0 & 0 \\ 0 & 0{,}0740 & 0 & 0 \\ 0 & 0 & 0{,}0060 & 0 \\ 0 & 0 & 0 & 0{,}0141 \end{bmatrix}. \qquad (6)$$

$$\mathbf{C} = \left[c_{ij}\right], \qquad c_{ij} = \frac{\partial^2 E_P}{\partial q_i . \partial q_j},$$

$$\mathbf{C} = \begin{bmatrix} c_1 & -c_1 & 0 & 0 \\ -c_1 & c_1 + c_{23}.r_2^2 + c_{32}.r_2^2 & -c_{23}.r_2.r_3 - c_{32}.r_2.r_3 & 0 \\ 0 & -c_{23}.r_2.r_3 - c_{32}.r_2.r_3 & c_3 + c_{23}.r_3^2 + c_{32}.r_3^2 & -c_3 \\ 0 & 0 & -c_3 & c_3 \end{bmatrix} = \qquad (7)$$

$$= \begin{bmatrix} 19938.6 & -19938.6 & 0 & 0 \\ -19938.6 & 28061.1 & -3847.5 & 0 \\ 0 & -3847.5 & 65257.1 & -63434.6 \\ 0 & 0 & -63434.6 & 63434.6 \end{bmatrix}.$$

$$\mathbf{B} = \left[b_{ij}\right], \qquad b_{ij} = \frac{\partial^2 F}{\partial \dot{q}_i . \partial \dot{q}_j},$$

$$\mathbf{B} = \begin{bmatrix} b_1 & -b_1 & 0 & 0 \\ -b_1 & b_1 + b_{23}.r_2^2 + b_{32}.r_2^2 & -b_{23}.r_2.r_3 - b_{32}.r_2.r_3 & 0 \\ 0 & -b_{23}.r_2.r_3 - b_{32}.r_2.r_3 & b_3 + b_{23}.r_3^2 + b_{32}.r_3^2 & -b_3 \\ 0 & 0 & -b_3 & b_3 \end{bmatrix} = \qquad (8)$$

$$= \begin{bmatrix} 5 & -5 & 0 & 0 \\ -5 & 5{,}0361 & -0{,}0171 & 0 \\ 0 & -0{,}0171 & 5{,}0081 & -5 \\ 0 & 0 & -5 & 5 \end{bmatrix}.$$

The general solutions of the system of differential equations (5) in the harmonious appearance of disturbing forces and initial conditions $t = 0$, $q(0) = q_0$, $\dot{q}(0) = \dot{q}_0$, written in matrix form, are

$$q(t) = \sum_{r=1}^{4} \frac{2}{g_r^2 + h_r^2}[\mathbf{G}_r\mathbf{M}\dot{q}(0) + (-\alpha_r\mathbf{G}_r\mathbf{M} + \beta_r\mathbf{H}_r\mathbf{M} \quad + \mathbf{G}_r\mathbf{B})q(0)]\,.e^{-\alpha_r t}.\cos\beta_r t +$$
$$+ \sum_{r=1}^{4} \frac{2}{g_r^2 + h_r^2}[\mathbf{H}_r.\mathbf{M}.\dot{q}(0) + (-\alpha_r.\mathbf{H}_r.\mathbf{M} - \beta_r.\mathbf{G}_r.\mathbf{M} + \mathbf{H}_r.\mathbf{B}).q(0)]\,.e^{-\alpha_r t}.\sin\beta_r t + \qquad \textbf{(9)}$$
$$+ \mathrm{Re}\{\sum_{k=0}^{n}\sum_{r=1}^{4} \frac{2}{g_r^2 + h_r^2}\,\frac{\alpha_r.\mathbf{G}_r + \beta_r.\mathbf{H}_r + i.k.\Omega.\mathbf{G}_r}{\omega_r^2 - k^2.\Omega^2 + i.2.k.\sigma_r.\omega_r.\Omega}\mathbf{Q}.e^{ik\Omega t}\}$$

where:

$$g_r = -2\alpha_r\left(\mathbf{V}_r^T.\mathbf{M}.\mathbf{V}_r - \mathbf{W}_r^T.\mathbf{M}.\mathbf{W}_r\right) - 4\beta_r\mathbf{V}_r^T.\mathbf{M}.\mathbf{W}_r + \mathbf{V}_r^T.\mathbf{B}.\mathbf{V}_r - \mathbf{W}_r^T.\mathbf{B}.\mathbf{W}_r;$$
$$h_r = 2\beta_r\left(\mathbf{V}_r^T.\mathbf{M}.\mathbf{V}_r - \mathbf{W}_r^T.\mathbf{M}.\mathbf{W}_r\right) - 4\alpha_r\mathbf{V}_r^T.\mathbf{M}.\mathbf{W}_r + 2\mathbf{V}_r^T.\mathbf{B}.\mathbf{W}_r;$$
$$\mathbf{G}_r = g_r\mathbf{L}_r + h_r\mathbf{R}_r; \quad \mathbf{L}_r = \mathbf{V}_r.\mathbf{V}_r^T - \mathbf{W}_r.\mathbf{W}_r^T;$$
$$\mathbf{H}_r = h_r\mathbf{L}_r - g_r\mathbf{R}_r; \quad \mathbf{R}_r = \mathbf{V}_r.\mathbf{W}_r^T + \mathbf{W}_r.\mathbf{V}_r^T.$$

V is the modal matrix; **W** – the matrix of the imaginary part of the natural vectors of the damping system; $p_r = -\alpha_r \pm i\beta_r$ – natural values; $u_r = v_r \pm iw_r$ – natural vectors; σ_r – relative damping coefficient; α_r – damping coefficient; β_r – frequency of free damping vibration;

$\alpha_r = \sigma_r.\omega_r$; $\beta_r = \omega_r\sqrt{1-\sigma_r^2}$; w_r – the imaginary part of the natural vector caused by dampening system; v_r, ω_r – natural modes and natural frequencies of the non damping system.

The developed mechanical - mathematical model allows conducting a number of studies of the forced torsional vibrations in the wood shaper's saw unit. Forced vibrations due to the variable moments on the drive electric motor and on the wood shaper's saw are investigated in this study. The unavoidable deviation of the correct shape of the stator and the unbalance of the rotor lead to the occurrence of a variable torsional moment on the electric motor. This moment is modeling as to its constant part are added two components that have the type $M_{11} \sin \omega_1 t$ and $M_{12} \sin 2\omega_1 t$ (where ω_1 is the frequency of rotation of the rotor; M_{11} and M_{12} are their amplitudes respectively).

The modeling of the variable moment of the wood shaper's saw demand studious analysis of the main features of the saw's work. The investigations, conducting by the authors, show that this moment can be given as to the concentrated saw's moment M_4 is added a variable component M_{4P}. This component of the moment is

$$M_{4P} = M_P \sin z\omega_4 t\,, \qquad \textbf{(10)}$$

where:

M_P is the amplitude of the variable component of the moment,

z – the blades' number of the saw, which really work,

ω_4 – the frequency of rotation of the shaper saw.

Numerical investigations are conducted for three different work regimes of the wood shaper's saw. In the first case it is accepted that all six blades of the saw take part in the saw process, т. е. $z = 6$. But practically only a part of the saw's blades work because there are some inaccuracies in the saw's production, their assembly and the cuter grinding. Two cases are investigated in this study– when the really working blades of the saw are three, i. e. $z = 3$, and when they are two, i. e. $z = 2$.

The moments M_2 and M_3 on pulleys from their interaction with the belt are taken into consideration.

RESULTS

The data of the wood shaper, which are necessary for the investigations, are given in the table.1

Table 1: Data 1

d_1 – diameter of the electric motor's shaft (mm)	28
d_3 – diameter of the spindle (mm)	44
r_2 – radius of the belt puller *2* (mm)	95
r_3 – radius of the belt puller *3* (mm)	45
l_1 – distance between the belt puller *2* and the electric motor (mm)	240
l_3 – distance between the shaper saw and the belt puller *3* (mm)	460
M_1 – moment of the electric motor (N.m)	9,554
M_{11} – additional moment of the electric motor (N.m)	4
M_{12} – additional moment of the electric motor (N.m)	4
M_2 – moment of the belt puller *2* (N.m)	0,2
M_3 – moment of the belt puller *3* (N.m)	0,15
M_4 – moment of the shaper saw (N.m)	5,6
M_P – additional moment of the shaper saw (N.m)	2.8
ω_1 – frequency of rotation (s^{-1})	46,82
ω_4 – frequency of rotation of the shaper saw (s^{-1})	628,32

The conducted investigations show that the influence of the blades' number of the saw, which really work, on the forced torsional vibrations of the drive electric motor and of the belt puller on the electric motor's shaft is slightly. But this influence is significantly on the second belt puller and it is bigger on the shaper's saw. The graphs, that illustrate the forced torsional vibrations of these two elements when all six blades of the saw take part in the saw process, are shown in fig. 4 and fig. 5. The respective graphs that illustrate vibrations when three blades work are shown in fig. 6 and fig. 7. Fig. 8 and fig. 9 illustrate vibrations when two blades work.

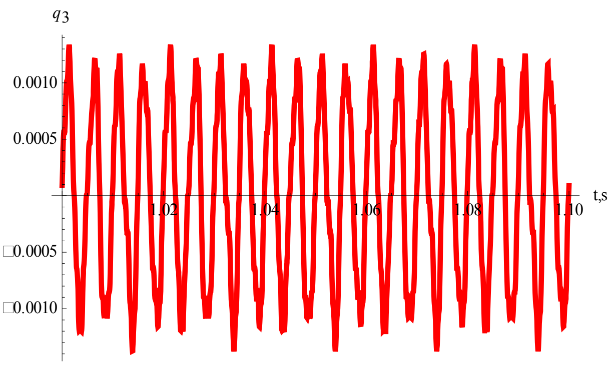


Figure 4:

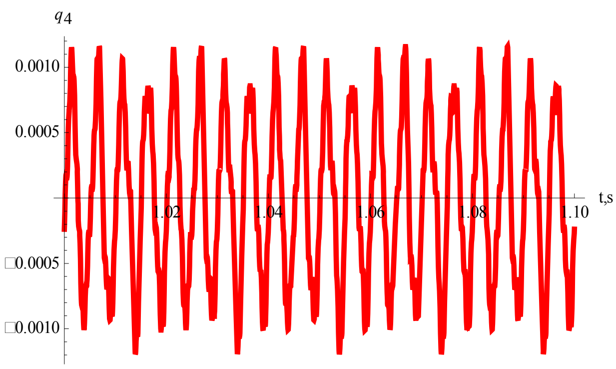


Figure 5:

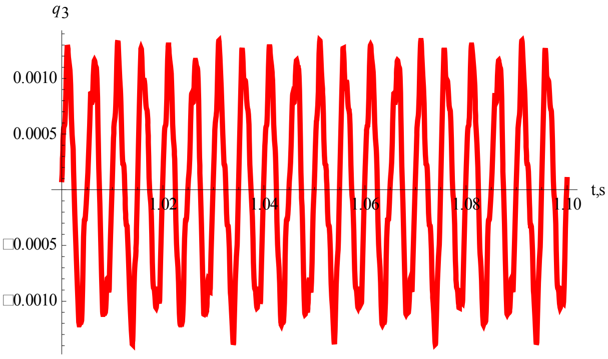
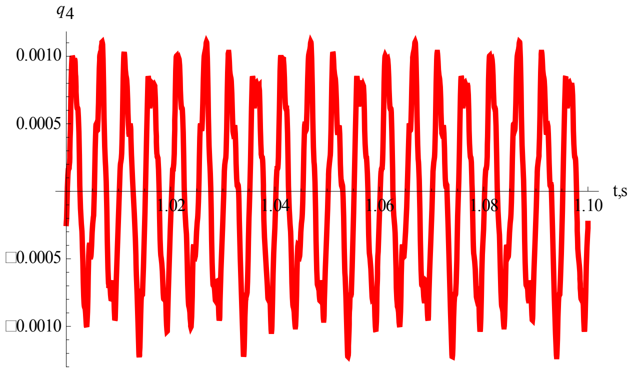


Figure 6:



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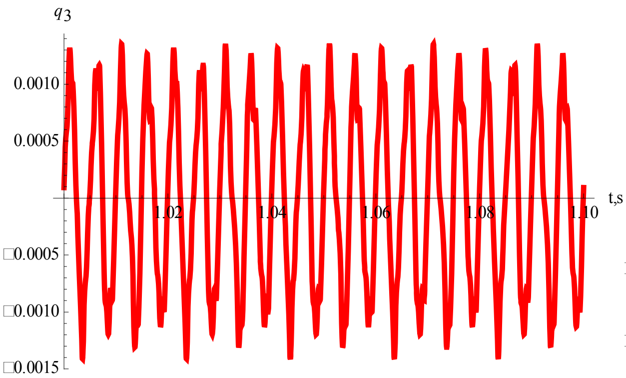

Figure 8:

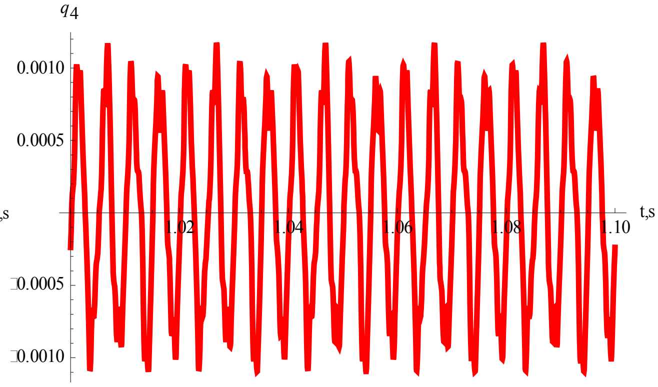

Figure 9:

Some deductions follow from figures above. The maximal amplitudes of the forced torsional vibrations of second belt puller increase when the blades' number of the saw, which really work, decrease. The increasing of the maximal amplitudes of the forced torsional vibrations of the shaper's saw is bigger. However, the increasing of the amplitudes of the single harmonics is strongly differently. The analysis of the obtained results shows that the blades' number of the saw, which really work, has a direct influence on the vibrations of the shaper's saw. This fact reflects on the accuracy and quality of machine's work.

CONCLUSION

The study presents the results of the implementation of numerical investigations with original mechanical mathematical model of the torsional vibrations of the wood shaper's saw unit. The forced torsional vibrations due to the variable moments on the drive electric motor and on the wood shaper's saw are examined. The influence on the vibrations of the blades' number of the saw, which really work, is investigated. The conducted analysis of the obtained results shows that the blades' number of the saw, which really work, has a direct influence on the vibrations of the shaper's saw unit. This influence has more weak effect on the vibrations of the second belt puller, but this effect on the vibrations of the shaper's saw is bigger. The increasing of the level of the vibrations inevitably makes the accuracy and quality of the production worse. The recommendations to supervision and control of the assembly and the technical state of the saw's blades are imposed. Thus guarantees the quality of the production.

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