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DEVELOPMENT OF A MODEL FOR THE VARIABLE TENSILE STRESS IN BAND-SAW BLADE

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ABSTRACT

As a continuation from a previous paper, an original model is proposed for the variable tensile stress in the blade (the saw band) which girdles the two wheels of a band-saw. The case of a spring mechanisms for tensioning the saw band is treated. The internally statically indeterminate problem of the closed band's contour is solved in a way as required by the course of Strength of Materials. At an instant position of the blade, the tensile stress in all the cross-sections along the band is searched for, taking into account: the assembly pre-tension; the cutting force; the bending of the saw band at the wheel periphery; the heating; the centrifugal inertial forces due to the curvilinear blade's motion as a new point in the development of the model proposed now. Another new, basic point is that the spring is not ideally stiff but "normal" now. An equation for the tensile internal force at an „opening" in the band is deduced by involving differential and integral analysis. The equation is solvable by iteration. In case of zero cutting force, the L'Hôpital's rule is applied.

Key words: band-saw blade, statically indeterminate closed contour, tensile force and stress along a saw band, centrifugal inertial forces

INTRODUCTION

As a continuation from a previous paper (Stefanov et al. 2012), the present paper represents a study aimed at establishing determination of stress in a band-saw blade in a way the course of Strength of Materials requires. It is meant that the saw band, the two wheels *and the spring mechanism* for tensioning the band build *a statically indeterminate system*. Separately, the *closed contour of the band is a bright example of internal statical indeterminacy* as already emphasized in (Stefanov et al. 2012).

In such a case, *a deformation compatibility condition* should be formulated for solving the statically indeterminate problem. In (Stefanov et al. 2012), a particular compatibility equation has been set which is valid for an ideally stiff spring. In this paper, a generalized compatibility condition is already formulated valid also for a "normal" spring. Now, the spring constant participates together with displacement of the spring's end pushed by a screw. Another new feature of the model is participation of centrifugal inertial forces.

CALCULATION SCHEME, COMPATIBILITY CONDITION AND *X*

Fig. 1 shows an original scheme from (Stefanov et al. 2012), now completed with participation of centrifugal forces and a generalized compatibility condition. An instantaneous kinetostatic position of the band is illustrated. The blade moves counterclockwise with a constant velocity *v*. The analysis begins with forming the external forces on the band.

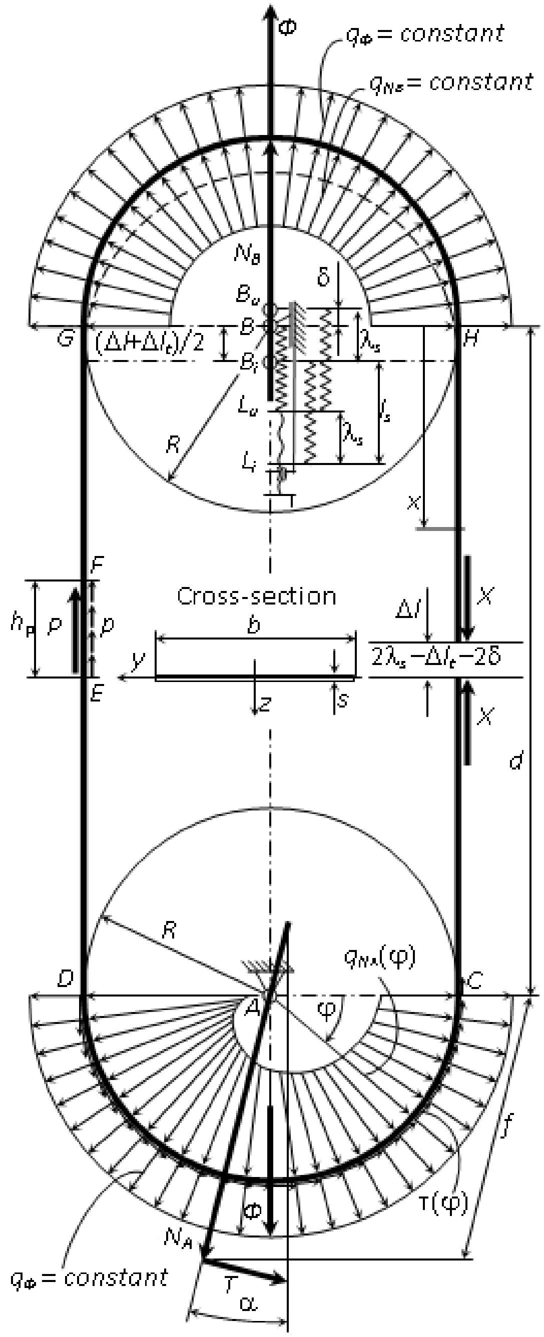

Figure 1: Calculation scheme

On the center B of the upper, driven wheel, a force N_B of the spring is exerted. It is transmitted to the band as a transversely distributed load of intensity q_{NB} (force per unit length). Stepwise appearance of q_{NB} at H, q_{NB} = *constant* between H and G, and again stepwise disappearance of q_{NB} at G are introduced in the model. This is an idealization entailed by the assumption that the branches GD and HC are ideally straight and transfer into exact upper and lower half-circumferences. Based on such idealization, the Euler problem of an ideally flexible and massless cord girdled a wheel is accepted as valid for a saw band (and for belt drives, as well). This problem will be discussed further below.

In reality, the saw band is not ideally flexible but has certain bending stiffness. The „straight" braches GD and HC cannot remain straight: the band curvature radius ρ cannot suddenly change from ∞ to R at H, and from R to ∞ at G. Such a sudden change in ρ while deflecting could be only due to a concentrated external moment causing a "jump" to the internal bending moment M_y (Stefanov 2007). However, there are not any external moments at H and G. Thus, ρ does not change suddenly but steeply and smoothly from ρ >> R to R at H, and from R to ρ >> R at G. Correspondingly, the intensity of pressing q_{NB} does not appear and disappear stepwise but smoothly, and does not remain constant. And, if the actual smooth ρ change (Mihaylov 1985) and the respective q_{NB} change are searched for, this will be a quite tough problem. It will concern additional deformation, strength and dynamic effects, which could be involved in further expanding the model. For now, the first, basis-laying thing to do is deduction of the equations for the basic, nominal internal forces and stresses along the saw band based on solving the statically indeterminate problem.

From the q_{NB} intensity, the imaginarily concentrated force $N_B = q_{NB}.2R$ results.

The lower, driving wheel presses the band with a distributed transverse force having intensity $q_{NA}(\varphi)$. Again, in an idealized way, this intensity appears and disappears stepwise at D and C. Between D and C, $q_{NA}(\varphi)$ is not constant now. The driving wheel moves the band by a tangentially distributed friction force with an intensity $\tau(\varphi)$. By concentrating $\tau(\varphi)$, an imaginary single force T (of traction) appears as a resultant at some distance f from the center A. The equation $T = \mu N_A$ holds where μ is a working coefficient of friction (coherence). This μ is not the sliding friction coefficient (Stefanov 2006 – 2012), although the same symbol is used here (for convenience), but is a part of the ultimate coefficient μ_0 of friction: $\mu \leq \mu_0$. If the coherence between the band and the wheel cannot provide $\mu \leq \mu_0$, then sliding will occur. The ratio $T/N_A = \mu$ is also tgα (Fig. 1).

Along both the upper and the lower half-circumference, an equal intensity q_Φ = *constant* of two transversely distributed centrifugal inertial forces $\Phi = q_\Phi.2R$ is stepwise introduced. In addition, a longitudinally distributed cutting force of intensity p acts along a segment EF of the working (the left) branch of the band. From p, an imaginary concentrated cutting force P results. For simplification, p = *constant* can be introduced, i.e. $p = P/h_p$ where h_p is the length of the EF band's segment (the height of the saw kerf).

The so-formed external forces on the saw band should fulfill the three conditions of (kinetostatic) equilibrium of the coplanar force system built. The two Φ forces will not participate in writing the conditions since they equilibrate each other. The moment equilibrium condition $\Sigma M_{i,A} = 0$ leads to $PR = Tf$. From the condition $\Sigma V_i = 0$ for the vertical forces, the equation $N_A\cos\alpha + T\sin\alpha = P + N_B$ results. Fulfilling the condition $\Sigma H_i = 0$ for the horizontal forces means $T/N_A = \mathrm{tg}\alpha$. Provided that P is a given force, the equilibrium equations have consecutively involved the following five loading parameters: T, f, N_A, α, and N_B. Further below, another equation will yield $\mu = \mathrm{tg}\alpha$. Thus, four parameters will have to be determined whereas the equilibrium equations are three: the problem is one time statically indeterminate.

At this point, it is to interpose that if N_B had been exerted by a lever-weight or hydraulic mechanism, this force would have assumed the role of a given, constant and independent load. *In that case, the problem of the tensile internal forces and stresses in a saw band becomes statically determinate* and much easier: neither assembly stresses nor thermal stresses appear in statically determinate systems under any dictates of any compatibility conditions (Stefanov 2007). Now, however, N_B depends on the spring deformation and its compatibility, and is therefore among the four loading parameters to determine.

Consideration of deformation components due to the different factors follows, as well as their combination in a compatibility condition for solving the statically indeterminate problem. The succession of appearance of the factors does not affect the result and therefore a sequence is to choose which makes the analysis easier. First, let the top wheel be in an initial lowest position B_i such that the saw band accepts the contour formed by the two semi-circumferences and the two straight branches. The upper semi-circumference is shown in Fig. 1 by a dashed line. Let the first factor be a temperature change Δt; $\Delta t > 0$ is heating, and $\Delta t < 0$ is cooling. Due to Δt, a free change in the band's length occurs: $\Delta l_t = \alpha_t l \Delta t$ (Stefanov 2007) where: α_t is the coefficient of thermal expansion (or compression if $\Delta t < 0$) per 1°C; $l = 2d + 2\pi R$ is the length of the saw band.

In the case of heating, the band lengthens by Δl_t and the top semi-circumference moves up. Let the top wheel follows it: B_i undergoes a free thermal displacement upwards. What is this displacement in comparison with Δl_t? The answer is simple: each displacement of the wheel's center upwards means lengthening the band that is twice as long. Indeed, this concerns the following easy geometrical problem: since $2\pi R$ is kept, then $\Delta(2d + 2\pi R) = \Delta(2d) + 0 = 2\Delta d$ where Δd is namely the mentioned displacement of the wheel's center upwards. Thus, the free thermal move-up of B_i is $\Delta l_t/2$.

Let the next factor is the appearance of all the forces counted above, including the N_B force. They all cause an elastic lengthening Δl_{el} which can be shortly labeled as Δl only. Following this Δl, the top wheel's center moves additionally upwards by $\Delta l/2$ to a working position B. The total center's displacement becomes $(\Delta l + \Delta l_t)/2$ (Fig. 1). At the same time, in the B position, the center is loaded by the counter-action N_B of the band in direction downwards, and by a spring force upwards. The two forces are in equilibrium, i.e. the spring force is again N_B.

The process of coming of the upper spring's end into the B position and the appearance of the spring force N_B can be presented in the following way. Let first the spring be free and not deformed (contracted): still not connected with B, and having length of spring l_s. The lower spring's end is at an initial level L_i. The screw shown in Fig. 1 pushes and moves up that spring's end at a distance λ_s. The spring is not deformed yet and keeps its length as shown in Fig. 1 (a bit to the right) by a thin line. The upper spring's end finds itself at a level B_u above B_i at a distance that is again λ_s. Let that upper end connect with B now. The spring is now contracted by δ and causes the force $N_B = c\delta$ where c is the spring constant.

Then, Fig. 1 shows the following compatibility equation: $(\Delta l + \Delta l_t)/2 + \delta = \lambda_s$.

Separately, the closed band's contour is internally statically indeterminate (Stefanov 2007): a single cross-section cannot divide it into two parts for finding the internal forces in the cross-section based on the equilibrium conditions of any of the two free-body diagrams. Thus, the internal forces in question remain statically indeterminate and, therefore, the statical indeterminacy is called internal. For solving the problem and finding the internal forces in a closed contour, it "is opened" first. In the case considered (Fig. 1), an „opening" is shown in the non-working (the right) straight branch of the band: two infinitely close cross-sections are imaginarily parted from each other and "are allowed" to draw wide apart. However, the two equal and opposite internal tensile forces N_x of the common magnitude X are set. This X should be determined in a way as to have the two cross-section connected back. The so-built system with the "opening" and X is called equivalent (Stefanov 2007).

As each displacement of the top wheel's center multiplied by two transforms into a change in the band's length, then the above condition $(\Delta l + \Delta l_t)/2 + \delta = \lambda_s$ is just multiplied by two. Hence, the compatibility condition searched for solving the statically indeterminate problem appears in the form $\Delta l + \Delta l_t + 2\delta = 2\lambda_s$. It is expedient to have it written as

$$\boldsymbol{\Delta l = \lambda - \Delta l_t - 2\delta} \qquad \textbf{(0)}$$

where $\lambda = 2\lambda_s$ is the double distance at which the screw pushes out the lower spring's end.

Eq. (0) can be interpreted in a way, as follows. First, the two cross-sections of the „opening" draw wide apart at an assembly distance λ due to the $2\lambda_s$ move of the pushed spring considered ideally stiff. Next, the λ

distance decreases by Δl_t due to the heating: the two cross-sections draw closer due to the thermal lengthening the band. Next, the spring becomes soft, contracts by δ, and the two cross-sections draw additionally closer by 2δ. To have the „gap“ between the two cross-sections completely closed, the two *X* forces, together with the tensile forces N_x in all the other cross-sections, cause the elastic lengthening Δl. In the previous paper (Stefanov et al. 2012), the compatibility equation was $\Delta l = \lambda - \Delta l_t$, i.e., the spring “remained” ideally stiff.

For development of Eq. (0), as well as for all the purposes of the next analysis, a current cross-section of abscissa *x* has to be made within every segment of the band’s length. The *x* abscissa has been accepted to begin from zero at the beginning of each of the segments *HC*, *CD*, *DE*, *EF*, *FG* and *GH*, and follow the band clockwise. Along the *HC* segment, *x* begins from *H* (Fig. 1), along *CD x* turns into a curvilinear abscissa which begins from *C*, and so on.

There is, in fact, a second statically indeterminate internal load, as well: the bending moment M_y due to the bending (the deflection) of the saw band along the two half-circumferences. In the case considered, the M_y moment is, however, determinable based on another compatibility condition: $M_y = EJ_y/\rho$ (Stefanov 2007) where ρ is the radius of the deflection curvature. From the idealization for exact half-circumferences with $\rho = R = constant$, $M_y = constant$ results. This bending moment is very little. In (Stefanov et al. 2012), in an calculation example with concrete data, M_y = 0,2083 N.m resulted. This is an insignificant bending moment, indeed: a saw band is (easily) flexible, though.

The insignificant bending moment causes, however, a significant stress of bending $\sigma_{be} = M_y/W_y$ since the cross-sectional moment of resistance $W_y = bs^2/6$ is also very little; $\sigma_{be} = M_y/W_y = [E(bs^3/12)/R]/(bs^2/6) = Es/(2R)$. This equation for σ_{be} can also be seen in (Atanasov 2012). In the mentioned calculation example, σ_{be} = 250 MPa was obtained. This is a high stress and therefore the saw bands are manufactured of complex alloyed steels of high ultimate strength in the order of 1000 and over 1000 MPa.

By the way, there is an engineering rule: $s \leq 2R/1000$ (Atanasov 2012). It results, in fact, from the strength condition $\sigma_{be} = Es/(2R) \leq \sigma_{adm}$ with $E = 2.10^{11}$ generally for steels and admissible stress σ_{adm} set to be 200 MPa. Modifications of this rule are based, in fact, on selecting a different σ_{adm} value.

2. DETERMINATION OF *X* AND THE INTERNAL TENSILE FORCE $N_X(X)$ ALONG THE WHOLE LENGTH OF THE SAW BAND

In Fig. 2, from the circumferential segment *CD* of the band, an infinitesimal portion is detached by two cuts at abscissae *x* and *x* + d*x*. The portion has a length d*x* and angular scope dφ. Its free-body diagram includes the two internal tensile forces N_x and $N_x + dN_x$, and the two equal internal bending moments M_y. Along the length of the portion d*x*, the following forces act: a normal compression force $dN \equiv dN_A$, a friction force d*T* and a centrifugal inertial force d*Φ*. Since d*x* is an infinitesimal length, the forces d*N*, d*T* и d*Φ* can be illustrated anywhere along d*x*, e.g. at the right end of the portion.

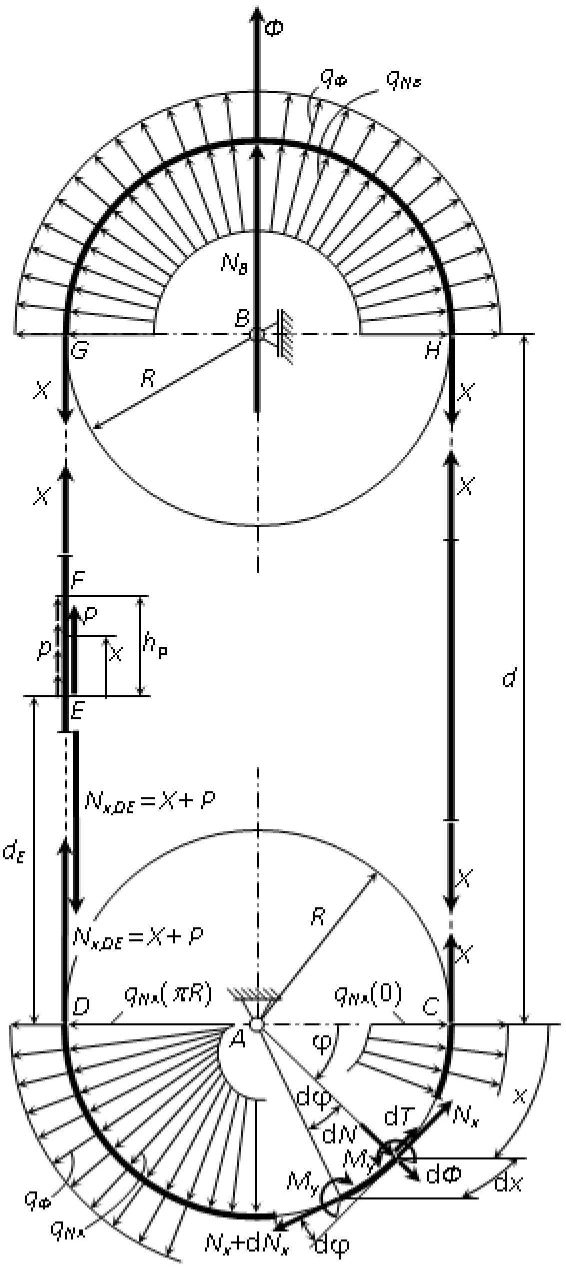

Figure 2: Illustration to the analysis

The force equilibrium equation in the radial direction at the angle φ is: $dN = (N_x + dN_x)\sin d\varphi - d\Phi = (N_x + dN_x)d\varphi - d\Phi = N_x d\varphi - d\Phi$ (where $\sin d\varphi \rightarrow d\varphi$, and $dN_x d\varphi$ is a negligible infinitesimal of second order). The force equilibrium equation in the tangential direction is: $N_x = -dT + (N_x + dN_x)\cos d\varphi$. Since $\cos d\varphi \rightarrow 1$, then ~~$N_x$~~ $= -dT$ + ~~N_x~~ $+ dN_x$, and the equation $dN_x = dT$ results. In it, $dT = \mu dN$ can be substituted: the ratio $T/N_A = \mu = \mathrm{tg}\alpha$ is kept as a ratio dT/dN, as well. Thus, $dN_x = \mu dN$ results. After substituting dN from above, $dN_x = \mu(N_x d\varphi - d\Phi)$ is obtained.

As for $d\Phi$, the portion's mass is $\rho bs.dx$ where ρ is the mass density of the material, and $bs.dx$ is the portion's volume. Then, $d\Phi = \rho bs.dx.v^2/R = \rho bs$~~$R$~~$d\varphi.v^2/$~~$R$~~ $= \rho bsv^2 d\varphi$. By the way, the q_Φ intensity is $q_\Phi = d\Phi/dx = \rho bsv^2/R$, and hence $\Phi = q_\Phi 2R = 2\rho bsv^2$. The above equation $dN_x = \mu(N_x d\varphi - d\Phi)$ takes the form $dN_x = \mu d\varphi(N_x - \rho bsv^2)$. It is equivalent to write $d(N_x - \Phi/2) = \mu d\varphi(N_x - \Phi/2)$, and thus $d(N_x - \Phi/2)/(N_x - \Phi/2) = \mu d\varphi$.

The definite integral of the right-hand side of the last differential equation, from the lower limit $\varphi = 0$ to the upper limit $\varphi = x/R$, yields $\mu x/R$. The indefinite integral of the left-hand side is $\ln(N_x - \Phi/2)$. In agreement with φ from 0 to $\varphi = x/R$, the definite integral should be taken from $X - \Phi/2$ to $N_x - \Phi/2$: $\ln(N_x - \Phi/2) - \ln(X - \Phi/2) = \mu x/R$. Hence, along the segment CD, the equation $N_x(x) = e^{\mu x/R}(X - \Phi/2) + \Phi/2$ applies.

As for the other band's segments, Fig. 2 shows that $N_x = X$ in the non-working band's branch remains to act the same along the whole length $FGHC$. And, for the segment EF, the equation $N_x(x) = X + p(h_p - x) = X + (P/h_p)(h_p - x)$ is valid. With that, $N_x(0) = X + P = N_{x,DE}$ where $N_{x,DE}$ comes from the segment EF as $N_{x,DE} = e^{\mu x/R}(X - \Phi/2) + \Phi/2$. Hence,

$$\mu = \mathrm{tg}\alpha = \frac{1}{\pi}\ln\left(1 + \frac{P}{X - \Phi/2}\right) \qquad \textbf{(1)}$$

where $\Phi/2 = \rho bsv^2$.

A systematic arrangement of the $N_x(x)$ equations along the segments follows:

$$CD: N_x(x) = e^{\mu x/R}(X - \Phi/2) + \Phi/2 \qquad \textbf{(2)}$$

$$DE: N_x = const = X + P \qquad \textbf{(3)}$$

$$EF: N_x(x) = X + (P/h_p)(h_p - x) \qquad \textbf{(4)}$$

$$FGHC: N_x = const = X \qquad \textbf{(5)}$$

With $v = \Phi = 0$, Eq. (2) yields $N_x(0) = X$ and $N_x(\pi R) = e^{\mu\pi}X$. Hence, $N_x(\pi R)/N_x(0) = e^{\mu\pi}$. This is, in fact, the well-known Euler equation for the ratio of the tensile forces in the two branches of a band girdling a wheel. The Euler problem leading to this equation is, in the present paper, a part of the problem of determining $N_x(x)$ along the segment *CD*.

Now, Δl can be formed as a sum of components from the segments *CD*, *DE*, *EF* and *FGHC*. The sum will be substituted in Eq. (0). The Δl components from the segments *DE* and *FGHC* where $N_x = constant = X$ are directly substituted with expressions in the form "N_x.(length of segment)/(*Ebs*)" (Stefanov 2007). The Δl components from the segments *CD* and *EF* where $N_x \neq constant$ take the form "[integral of $N_x(x)\mathrm{d}x$]/(*Ebs*)" (Stefanov 2007). To solve the corresponding two integrals is easy: exponential and linear functions are involved. Besides, $\delta = N_B/c$ should also be substituted in Eq. (0) in order to involve the spring constant c which will actually be given instead of δ. In addition, N_B is to be substituted after its coming from the equilibrium equation of the forces on the upper wheel (Fig. 2): $N_B = 2X - \Phi$.

After all the substitutions and mathematical deductions, the following equation results:

$$X = \frac{1}{2d + \pi R + \dfrac{4Ebs}{c}}\left[-P\left(d_E + \frac{h_p}{2}\right) - \frac{PR}{\mu} - \frac{\pi R\Phi}{2} + Ebs\lambda - Ebsl\alpha_t\Delta l + \frac{2Ebs\Phi}{c}\right] \quad \textbf{(6)}$$

where $\Phi = 2\rho bsv^2$. With $v = \Phi = 0$ and $\delta = 0$, respectively $c \to \infty$ (ideally stiff spring), the above equations take the form they have in the previous paper (Stefanov et al. 2012).

The X force cannot be directly calculated: Eq. (6) is not directly solved for X since it participates also in the right-hand side via μ (behind the 'ln' sign in Eq. (1)). Instead, Eq. (6), so presented, allows *practical calculation of X by means of fixed-point iteration* using the following algorithm. An initial assumed $\mu = \mathrm{tg}\alpha$ is entered and, using Eq. (6), an initial X is calculated. A new μ is calculated by Eq. (1), and the calculation cycle is repeated. The iteration will conclude when the old and the new μ equalize (to the third or fourth significant figure). Such an algorithm is easy to organize by Excel, for example. The author can provide a respective Excel file to everybody concerned.

After X and μ are found from Eqs. (6) and (1), the internal tensile forces N_x along all the segments of the band become already determinable by their Eqs. (2) – (5). As well, $N_B = 2X - \Phi$ can be calculated. In addition, according to the equilibrium equations in chapter 1, the following equations for N_A, T, and the moment arm f of the T force (Fig. 1) are obtained: $N_A = (P + N_B)\cos\alpha$ where $\alpha = \mathrm{arctg}\mu$, $T = (P + N_B)\sin\alpha$, and $f = PR/T$ (it is not R).

3. THE TENSILE FORCE $X \equiv X_0$ IN THE CASE $P = 0$

The case $P = 0$ means idle running of the band (no cutting). According to Eq. (1), $\mu = \mathrm{tg}\alpha = 0$: the friction force T in Fig. 1 is absent, and the N_A force takes vertical position and a magnitude $N_A \equiv N_{A,0} = N_B \equiv N_{B,0} = 2X_0 - \Phi$. The same tensile force $N_x = constant = X \equiv X_0$ settles along the whole band's length. It will be calculated using Eq. (6) after setting $P = 0$ there. However, it is to take into consideration that the addend PR/μ

in the brackets transforms into the indeterminate form [0/0] (this issue was not perfected in (Stefanov et al. 2012)).

For revealing this indeterminate form, the well-known L'Hôpital's rule applies: the numerator and denominator of PR/μ are substituted with their derivatives with respect to P. The derivative of PR is R. The derivative of μ, after differentiating Eq. (1) with respect to P and then substituting 0 for P, proves $1/[\pi(X_0 - \Phi/2]$. Thus, it turns out that $PR/\mu \rightarrow [0/0] \rightarrow \pi R(X_0 - \Phi/2)$. Then, Eq. (6) yields $X \equiv X_0 = [1/(2d + \pi R + 4Ebs/c)][-\pi R(X_0 - \Phi/2) - \pi R\Phi/2 + Ebs\lambda - Ebsl\alpha_t\Delta t - 2Ebs\Phi/c]$. Here, $\pi R\Phi/2$ cancels out, and the final result is:

$$X_0 = \frac{1}{l + \dfrac{4Ebs}{c}}\left[Ebs\lambda - Ebsl\alpha_t\Delta t + \frac{2Ebs\Phi}{c}\right] \quad \textbf{(7)}$$

This equation can be represented in the form $X_0 = X_M - X_t + X_\Phi$ with meanings of the addends, as follows. With $\Delta t = 0$ and $\Phi = 0$, Eq. (7) yields the assembly (<u>m</u>ounting) tension: $X_M = Ebs\lambda/(l + 4Ebs/c)$. Hence, the necessary value of $\lambda = 2\lambda_s$ for enabling desired tension X_M is $\lambda = X_M(l + 4Ebs/c)/(Ebs)$. With starting the engine and appearing Φ, $X_\Phi = 2Ebs\Phi/(cl + 4Ebs)$ is added to X_M. And with appearance of Δt, $X_t = Ebsl\alpha_t\Delta t/(l + 4Ebs/c)$ is subtracted.

In the borderline case of $c \rightarrow \infty$, respectively $\delta \rightarrow 0$, the equations $X_M = Ebs\lambda/l$, $X_\Phi = 0$, and $X_t = Ebs\alpha_t\Delta t$ take place. In other words, in case of a very stiff spring, the appearing centrifugal forces while setting the band in motion do not influence the mounting tension which is $X_M \approx Ebs\lambda/l$. This means that with lengthening the band by the centrifugal forces and supervened loosing the spring, the intensity q_{NB} decreases by as much as the appearing intensity q_Φ is. In addition, the heating Δt leads to a decrease (a subtrahend) X_t of X_M, and this X_t converges to a maximum value $Ebs\alpha_t\Delta t$.

Let the other borderline case of $c \rightarrow 0$, i.e. of an infinitely soft spring, be also studied. For this purpose, it is pertinent to represent Eq. (7) in the following form:

$$X_0 = \frac{1}{cl + 4Ebs}\left[cEbs\lambda - cEbsl\alpha_t\Delta t + 2Ebs\Phi\right]. \quad \textbf{(8)}$$

Now, the subtrahend containing Δt becomes zero due to $c \rightarrow 0$. However, the minuend $cEbs\lambda$ does not become zero since it is of the form $[0.\infty]$. Indeed, $\lambda \rightarrow \infty$ is necessary for the minuend to produce a desired mounting tension X_M. Hence, the difference $\lambda_s - \delta = (\Delta l + \Delta l_t)/2$ becomes insignificant in comparison to λ_s and δ: $\delta \rightarrow \lambda_s$, i.e. $\delta \rightarrow \infty$, as well. Of course, then the free spring's length l_s should also be $l_s \rightarrow \infty$ in order to have sufficient length $l_s - \delta$ left after the contraction. After all, $X_M \rightarrow c\lambda/4 = c\lambda_s/2$, $X_\Phi = \Phi/2$ and $X_t = 0$. This means that with a very soft spring the appearing centrifugal forces increase the mounting tension $X_M \approx c\lambda_s/2 = c\lambda/4$ by an addition which converges to a maximum value $\Phi/2 = \rho bsv^2$. And, the subtrahend due to the heating Δt drops out.

Thus, it turns out that for keeping the mounting tension without weakening influence of the heating, and for strengthening this tension by the centrifugal forces, a soft spring is more appropriate than a stiff one. Then, the system converges to statical determinacy like with using a lever-weight or hydraulic mechanism. However, for enabling a desired mounting tension $X_M \approx c\lambda_s/2$ kept by a soft spring, the very spring, λ_s, and respectively the carriage of the upper wheel, should be long enough.

4. THE VARIABLE STRESS σ ≡ σ_X ALONG THE SAW BAND AND APPROXIMATE OSCILLOGRAM σ(*T*)

Technically, the preliminary mounting tension of the band $X_M = Ebs\lambda/(l + 4Ebs/c)$ is associated with a certain recommended assembly stress σ_M: $X_M = \sigma_M bs$ (Atanasov 2012). Thus, the necessary λ value is $\lambda = \sigma_M bs(l + 4Ebs/c)/(Ebs) = \sigma_M(l + 4Ebs/c)/E$. This expression can substitute λ in Eqs. (6) – (8). And the necessary move of the spring pushed out is $\lambda_s = \sigma_M(l + 4Ebs/c)/(2E)$.

Eqs. (2) – (5), after dividing by the cross-sectional area $A = bs$, transform into equations for the tensile stress $\sigma_{Nx}(x) \equiv \sigma_{x,Nx}(x) = N_x(x)/(bs)$. As to the stress σ_z due to q_{NA} and q_{NB}, it is insignificant (Stefanov et al. 2012): the state of stress remains uniaxial with $\sigma \equiv \sigma_x$.

It is to emphasize again, as said in (Stefanov et al. 2012), that while any cross-section of the running saw band is traveling the whole length *l*, the graph of the function $\sigma(x) = N_x(x)/A \pm \sigma_{be}$ is turning into a convenient oscillogram σ(*t*). In fact, the $N_x(x)$ diagram is „unfolded“ against the time *t*, the N_x ordinates are divided by $A = bs$, and $\pm\sigma_{be} = \pm Es/(2R)$ is added within the segments *HG* and *DC*. The stress σ_{be} appears with the plus sign at the external side of the band (at $z = -s/2$), and with the minus sign at the internal side (at $z = s/2$).

Based on all the analysis up to here, the oscillogram σ(*t*) during one period *T* has been plotted (Stefanov et al. 2012) as shown in Fig. 3 (see the solid line). One lap is followed: traveled by a cross-section starting upwards from the position *C* (Fig. 2). The maximum 270 MPa (shown as an example in the parentheses) refers to BU400 bandsaw. The oscillogram is valid for any point of the external band's side where the σ(*t*) variations keep the same plus sign.

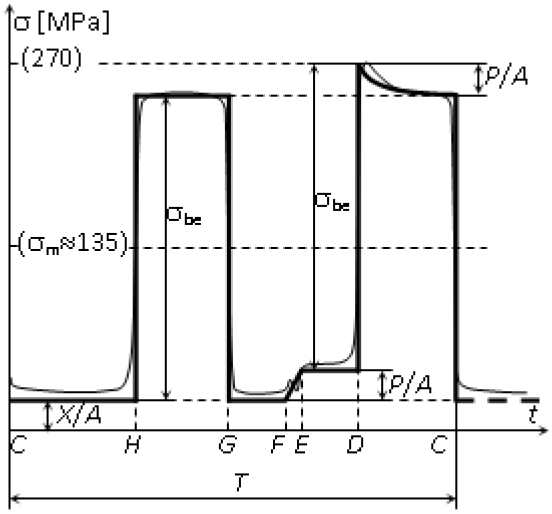

Figure 3: Approximate oscillogram σ(t)

In a next possible generalization and complication of the model, the following additional factors can be involved: a smooth transition from curvature radius ρ = *R* to ρ >> *R* and vice versa; vibration of the *B* center of the top wheel; transverse oscillations of the band; the *p* distribution intensity of *P* can be set as randomly changeable along h_p; etc. Such additional effects will change to some extent the shown basic σ(*t*) variations (see the solid line) and will superpose additional waves in them. A thin line in Fig. 3 implies a presumed approach to an actual oscillogram. However, anyway, the most essential variations of the oscillogram are due to the appearance and disappearance of σ_{be}.

In a more expanded treatment, the state of stress should not be left as uniaxial with $\sigma \equiv \sigma_x$ but, due to $s << b$, σ_y should also be involved according to the theory of bending of (thin) plates. There are other more, additional sources of stresses. In (Atanasov 2012), the influence of a band-guide wheel slope is discussed. It is to add here that this slope would cause torsion of the band and appearance of shear stresses $\tau(t)$. Separately considered, a concentrated multiaxial state of stress arises at the cutting teeth (Atanasov et al. 2012). In general, both a $\sigma_y(t)$ and a $\tau_{xy}(t)$ oscillogram will be involved. This all requires studying with regard to fatigue of the saw band material. More on this issue can be red in (Stefanov et al. 2012).

CONCLUSION

The deformation compatibility Eq. (0) has been formulated and the statically indeterminate problem has been solved as the Strength of Materials science requires. This should be presented in the foreground in the textbooks treating the internal forces and stresses in a saw band. However, such a treatment is missing in (Filipov 1977, Obreshkov 1995) and other textbooks used for teaching the students at the Faculty of Forest Industry (FFI). Respectively, the author hopes that this paper is an essential innovation for FFI.

It is seen that a non-simple mathematical expression, Eq. (6), is obtained for X and is carried into Eqs. (2) – (5) for the internal tensile forces along the band's segments. Any different way searched for determination of the tensile forces in question, based on any combination of components that are not associated with the compatibility Eq. (0), would be groundless. In addition, if vibration of a mass connected with the B point is studied, a lack of considering Eq. (0) would make such a study groundless, either. For that purpose, the proposed model should be used again after adding now the inertial force the upper wheel exerts on the band due to the oscillatory motion $\delta = \delta(t)$.

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