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NUMERICAL INVESTIGATIONS OF THE NATURAL FREQUENCIES AND MODE SHAPES OF THE TORSIONAL VIBRATIONS IN THE SAW UNIT OF A KIND OF WOOD SHAPERS, USED IN THE WOOD PRODUCTION

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ABSTRACT

Some numerical investigations of the natural frequencies and mode shapes of the wood shaper's saw unit are presented in this study. These researches are done on the base of a concrete mechanic-mathematical model for investigation of free torsional vibrations of a wood shaper's saw developed by the authors. The main features in the construction of these kind wood shapers are rendered an account of this model. The two most commonly used driving mechanisms are modeled – with a wedge belt and with a ribbed belt. The studies allow a comparison of the vibration behavior of the mechanism for both drives. The results of this study allow the determination of the resonant work regimes. The exactly determination of these regimes is important for introduction of adequate measures which can guarantee their using. The investigation`s results can be used as a base for making some recommendations concerning the increase of reliability of the wood shapers as well as the accuracy and quality of their production.

Key words: wood shapers, modeling, torsional vibrations

INTRODUCTION

The enhanced requirements for reducing the level of vibration and noise accompanying the operation of modern woodworking machines significantly improve significantly the study of dynamic processes in them. It mainly concerns wood shapers which are machines with a high level of vibration and noise. Discovering the causes for originate and increase of the vibration and noise level requires understanding the essence of the phenomena characteristic of the machine and its individual elements (Veits at al. 1971). It is necessary to conduct concrete studies in which the machine can be considered as a mechanical vibrating system with known characteristics (Amirouche 2006).

The main task of the research – reducing the level of the vibration and noise – needs to formulate concrete measures and ways to influence the vibrating system. This in turn requires the introduction of specific requirements for the construction and operation of its elements (Georgieva and Dichev 2011). For this purpose, firstly it is necessary to have mechanical-mathematical modeling and composing of equations describing the vibration of the elements of the wood shaper. Well-targeted research can be done by solving these equations in different conditions. Some recommendations for the construction's design and the work regimes of the machine are formed on their base.(Vukov at al. 2010).

The kind of wood shapers that are commonly used in the practice of forestry industry (Filipov 1977) are examined in the proposed study. Fig. 1 shows the general view, and Fig. 2 – a scheme of this type of wood shapers (Obreshkov 1997). The machine body is marked with *1*, *2* is the electric motor, *3* – the belt drive, which can be a wedge belt or a ribbed belt, *4* – the spindle with the bearings, *5* – the abror with morse cone, *6* – the work table, *7* – the cutter.

Figure 1: Wood shaper general view

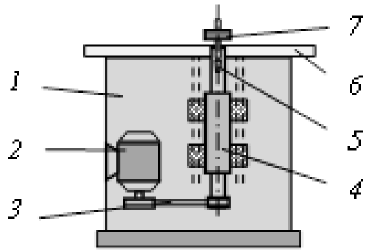

Figure 2: Scheme of the wood shaper

The next figures show the components of the wood shaper's saw unit that is the subject of this study. Fig. 3 shows the electric motor with a pulley, attached to its shaft, and Fig. 4 – the spindle and its bearings. Fig. 5 shows the cutter.

The vibration characteristics of the wood shaper's saw units depend on their natural vibration frequencies as well as any mechanical vibration system. It is known that when the frequencies of the external influences, which cause vibrations, are equal to a frequency of their natural frequencies appears the phenomenon "resonance". Resonance regimes can lead to significant increase of vibration amplitudes. Significant vibration amplitudes change the normal work regimes of the wood shaper's saw unit and damage the accuracy and quality of the production (Vlasev and Vukov 2002). Extra stress, which is caused by increase of vibration amplitudes, sometimes can reach such values that can damage or even destroy some machine's elements. (Marinov 2005) This causes reduction of the reliability of the wood shaper.

Figure 3: Electric motor with a pulley

Figure 4: Spindle and its bearings

Figure 5: Cutter

A cardinal rule in the engineer's practice is that resonant effects are unwilling and they must be avoided by a suitable selection of parameters of the wood shaper's saw units and other details and units of the machine, as well as the work regimes (Coutinho 2001). Therefore, it is necessary to make in advance an evaluation of the resonant danger when the wood shaper is designed and dimensioned. It does not allow danger work regimes during the operation exploitation. To solve this problem it is a must to study the natural frequencies. In cases that the resonant danger is available, some changes in the construction of the wood shaper or in the work regimes are advisable.

The aim of this study is to make a numerical investigation of the natural frequencies and mode shapes of the wood shaper's saw unit. The investigation is done on the base of an adequate mechanic-mathematical model for investigation of free torsional vibrations of the wood shapers developed by the authors. The model presents features in the construction of a kind of wood shapers. Two most commonly used driving mechanisms are modeled – with a wedge belt and with a ribbed belt. The studies allow a comparison of the vibration behavior of the mechanism for both drives. Some recommendations concerning the prevention from the resonant work regimes can be made on the base of this study. It is connected with the increase of reliability of the wood shapers as well as with the accuracy and quality of the production.

MECHANIC-MATHEMATICAL MODEL

The mechanic-mathematical model for investigation of the dynamical processes and vibrations in the wood shaper's saw unit is built by the authors. The model is shown on the fig. 6. This model includes four discrete mass connected with three massless elastic elements. φ_i, $i = 1, 2, 3, 4$ are the angles of the rotation of the corresponding rotor. The elasticity coefficients of the electric motor's shaft, the belt and the spindle are taken into account. The elasticity angular coefficient of the electric motor's shaft is marked with c_1, and this one of the spindle – with c_3 (*N.m/rad*). The elasticity linear coefficients of the two parts of the belt between the belt puller are c_{23} and c_{32} (*N/m*).

The necessary reduced mass inertia moments ($kg \cdot m^2$) render in account (fig. 6):

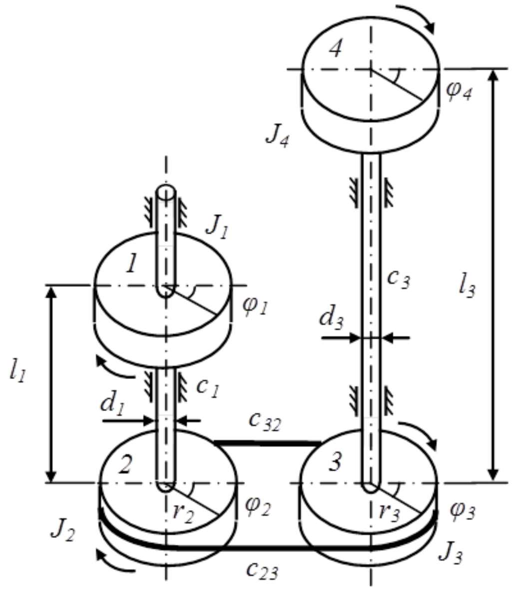

Figure 6: Mechanic-mathematical model

J_1 – the mass inertia moment of the electric motor's rotor;

J_2 – the mass inertia moment of the belt puller on the electric motor's shaft;

J_3 – the mass inertia moment of the belt puller on the spindle;

J_4 – the mass inertia moment of the wood shaper's saw with cutter arbor.

The other some symbols on fig. 6 are:

d_1, d_3 – diameters of the electric motor's shaft and spindle (m);

l_1, l_3 – computing length of the electric motor's shaft and spindle (m);

r_2, r_3 – radius of the belt pullers on the electric motor's shaft and spindle (m);

G – modulus of shearing.

The investigation of the torsional vibrations of the wood shaper's saw unit requires formulation and solution of the differential equations which describe these processes. Therefore, the priority of the matrix mechanics is used (Angelov and Slavov 2010).

The mechanic-mathematical model is done by using the applied engineer program (Mathematica). An algorithm for formulation of the matrixes which describe the mass-inertial properties and the elastic properties of the mechanical system is developed. The differential equations which describe the free vibrations are deduced by using the Lagrange's method.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right)-\frac{\partial E_K}{\partial q}+\frac{\partial E_P}{\partial q}=0\,, \tag{1}$$

where q_i are the generalized coordinates, E_K and E_P are respectively the kinetic and the potential energy of the multibody systems.

The vector of the generalized coordinates is

$$\mathbf{q}=\left[\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4\right]^T. \tag{2}$$

The kinetic energy of the mechanical system is obtained as a sum of the kinetic energy of the four basic bodies (the electric motor's rotor, the belt puller on the electric motor's shaft, the belt puller on the spindle, wood shaper's saw). By analogy the potential energy of the mechanical system is obtained as a sum of the potential energies received from the deformations of the electric motor's shaft, the belt and the spindle

$$E_K=\frac{1}{2}J_1.\dot{\varphi}_1^{\,2}+\frac{1}{2}J_2.\dot{\varphi}_2^{\,2}+\frac{1}{2}J_3.\dot{\varphi}_3^{\,2}+\frac{1}{2}J_4.\dot{\varphi}_4^{\,2}\,,$$

$$\begin{aligned}E_P&=\frac{1}{2}c_1.(\varphi_1-\varphi_2)^2+\frac{1}{2}c_{23}.(r_2.\varphi_2-r_3.\varphi_3)^2+\\&+\frac{1}{2}c_{32}.(r_3.\varphi_3-r_2.\varphi_2)^2+\frac{1}{2}c_3.(\varphi_3-\varphi_4)^2\end{aligned}\,. \tag{3}$$

The obtained system of parametric differential equations which describes the free torsional vibrations of the mechanical system is

$$\mathbf{M}.\ddot{\mathbf{q}}+\mathbf{C}.\mathbf{q}=0\,. \tag{4}$$

The matrix, which characterizes the mass-inertial properties **M** and the elastic properties **C** of the mechanical system, are

$$\mathbf{M}=\left[a_{ij}\right],\qquad a_{ij}=\frac{\partial^2 E_K}{\partial \dot{q}_i\,.\partial \dot{q}_j},\qquad \mathbf{M}=\begin{bmatrix}J_1 & 0 & 0 & 0\\ 0 & J_2 & 0 & 0\\ 0 & 0 & J_3 & 0\\ 0 & 0 & 0 & J_4\end{bmatrix}, \tag{5}$$

$$\mathbf{C}=\left[c_{ij}\right],c_{ij}=\frac{\partial^2 E_P}{\partial q_i\,.\partial q_j},\mathbf{C}=\begin{bmatrix}c_1 & -c_1 & 0 & 0\\ -c_1 & c_1+c_{23}.r_2^{\,2}+c_{32}.r_2^{\,2} & -c_{23}.r_2.r_3-c_{32}.r_2.r_3 & 0\\ 0 & -c_{23}.r_2.r_3-c_{32}.r_2.r_3 & c_3+c_{23}.r_3^{\,2}+c_{32}.r_3^{\,2} & -c_3\\ 0 & 0 & -c_3 & c_3\end{bmatrix}$$

Particular solutions to the system of the differential equations (4) are searched as

$$q_r=h_r.\sin\left(\omega_r.t+\varphi\right), \tag{6}$$

where h_r is the amplitude of the small vibration on the generalized coordinate q_r with natural frequency ω_r, and φ is the initial phase.

After differentiation of (6) and substituting in (4) it obtains a system of linear algebraic equations. In the matrix description they are:

$$\left|\mathbf{C}-\omega^2.\mathbf{M}\right|.\mathbf{V}=0\,. \tag{7}$$

To determine the natural frequencies and the mode shapes, it is necessary to solve the task about finding the natural values and the natural vectors of the equations (7).

The satisfaction of the equations (7) requires the following

$$\det\left|\mathbf{C}-\omega^2.\mathbf{M}\right|=0\,. \tag{8}$$

The roots of the characteristics equation determine the natural frequencies. The natural frequencies form the matrix of the natural values. They are

$$\omega = \mathrm{diag}\left\lfloor \omega_{r,r} \right\rfloor, \qquad i = 1,\ 2,...,4\,. \qquad \textbf{(9)}$$

The natural frequencies [*Hz*] are determinated by (9)

$$f_r = \frac{\omega_{r,r}}{2\pi} \quad \mathrm{Hz}\,. \qquad \textbf{(10)}$$

The natural values of the system (8) determine the natural vectors of the mechanical system.

A natural vector $\mathbf{v_r}$, which gives correlation between amplitudes of the vibrations, corresponds to every natural frequency ω_r. The vector's components define the matrix of the natural vectors (modal matrix) of the system (7) that is

$$V = \left\lfloor v_{r,j} \right\rfloor_{4\times 4}\,, \qquad \textbf{(11)}$$

where $\mathbf{v_r} = \left\lfloor v_{r,1} \quad v_{r,2} \quad v_{r,3} \quad v_{r,4} \right\rfloor$ is the natural mode vector on the generalized coordinate for r^{-th} natural frequency.

RESULTS

Some investigations of the wood shaper's saw unit with both drives: first – with a wedge belt and second – with a ribbed belt are carried out. The mass, elastic and geometrical characteristics of the elements of the wedge-belt mechanism are shown in the Table 1, and characteristics of the ribbed-belt mechanism – in Table 2.

Table 1. Characteristics of the elements of the wedge-belt mechanism

J_1 – inertia moment of the electric motor's rotor ($kg{\cdot}m^2$)	0, 0102
J_2 – inertia moment of the belt puller *2* ($kg{\cdot}m^2$)	0, 0740
J_3 – inertia moment of the belt puller *3* ($kg{\cdot}m^2$)	0, 0060
J_4 – inertia moment of the shaper saw ($kg{\cdot}m^2$)	0, 0141
c_1 – stiffness of the electric motor's shaft (Nm/rad)	14016
c_2 – stiffness of the spindle (Nm/rad)	10324
c_{23} – stiffness of the belt (N/m)	$4{,}5.{\cdot}10^5$
c_{32} – stiffness of the belt (N/m)	$4{,}5.{\cdot}10^5$
d_1 – diameter of the electric motor's shaft (mm)	30
d_3 – diameter of the spindle (mm)	44
r_2 – radius of the belt puller *2* (mm)	190
r_3 – radius of the belt puller *3* (mm)	88
l_1 – distance between the belt puller *2* and the electric motor (mm)	240
l_3 – distance between the shaper saw and the belt puller *3* (mm)	460

Table 2. Characteristics of the elements of the ribbed-belt mechanism

J_1 – inertia moment of the electric motor's rotor ($kg{\cdot}m^2$)	0, 0102
J_2 – inertia moment of the belt puller *2* ($kg{\cdot}m^2$)	0, 0729
J_3 – inertia moment of the belt puller *3* ($kg{\cdot}m^2$)	0, 0048
J_4 – inertia moment of the shaper saw ($kg{\cdot}m^2$)	0, 0141
c_1 – stiffness of the electric motor's shaft (Nm/rad)	14016
c_2 – stiffness of the spindle (Nm/rad)	10324
c_{23} – stiffness of the belt (N/m)	$5.{\cdot}10^5$
c_{32} – stiffness of the belt (N/m)	$5.{\cdot}10^5$
d_1 – diameter of the electric motor's shaft (mm)	30
d_3 – diameter of the spindle (mm)	44
r_2 – radius of the belt puller *2* (mm)	190
r_3 – radius of the belt puller *3* (mm)	88
l_1 – distance between the belt puller *2* and the electric motor (mm)	240
l_3 – distance between the shaper saw and the belt puller *3* (mm)	460

The calculations are done with help of the applied engineer program Mathematica (www.mathematica.com). The natural frequencies [s^{-1}] (and in [min^{-1}], [Hz]) for two investigations are respectively
`1875.8; 1264.54; 706.65; 0; (17912.5; 12075.5; 6748.5; 0), (298.54; 201.26; 112.47; 0).`
`2100.43; 1268.53; 732.78; 0; (20057.6; 12113.6; 6997.54; 0) (334.29; 201.89; 116.63; 0).`

The calculated natural frequencies [*Hz*] and mode shapes of the torsional vibrations of the two types of mechanism research are illustrated graphically on fig. 7.

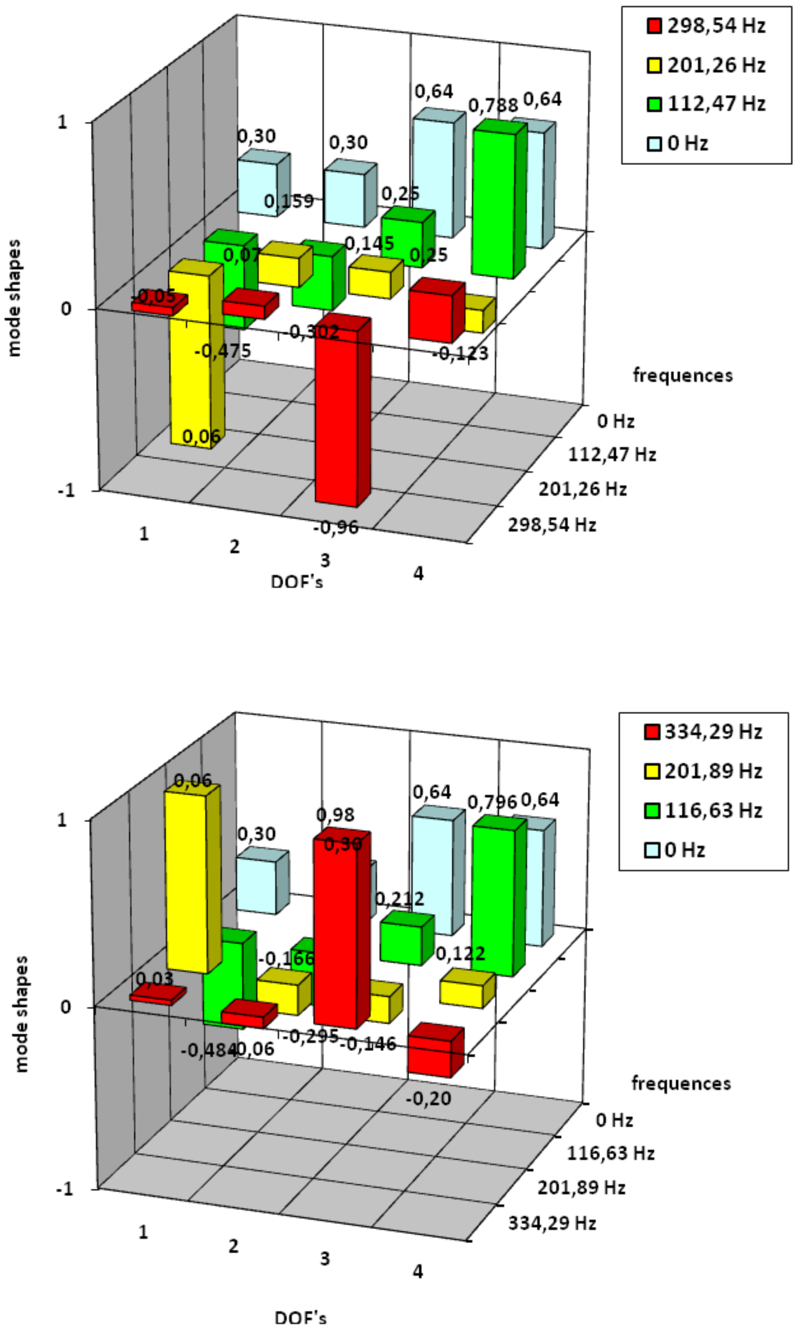

Figure 7: Mode shapes of the torsional vibrations of the two types of mechanism

The obtained results indicate the inevitable change of natural frequencies and mainly their mode shapes of the researched mechanism when the wedge belt is replaced with a ribbed belt. This is particularly important in analyzing the whole vibration behavior of the machinery.

CONCLUSION

This study presents a numerical investigation of the torsional vibrations of the wood shaper's saw unit. The natural frequencies and mode shapes are obtained and illustrated. The study is done numerically by using modern software. Calculations are made on the base of the specific mechanic – mathematical model developed by the authors for analyzing the torsional vibrations of a kind of the wood shapers. The advantages of the model are in render an accaunt of the characteristics of the construction of the class wood shapers and the ability to analyze and compare the vibration behavior of the system using a different type of belt drives. The results of this study allow formation of some recommendations for the avoidance of resonance regimes. The obtained results can be used as a base concerning the increasing of the reliability of the machine, accuracy and quality of processing products.

REFERENCES

1. Amirouche, F., (2006). Fundamentals of Multibody Dynamics – Theory and Applications, Birkhäuser, Boston.
2. Angelov, I., Slavov, V. R., (2010). Book of problems. Matrix mechanics - Dynamics, Avangard Prima, Sofia.
3. Coutinho, M., (2001). Dynamic Simulations of Multibody Systems, Springer-Verlag, New-York.
4. Filipov, G., (1977). Woodworking machines. Sofia.
5. Georgieva, D., P. Dichev. (2011). Economic Effectiveness of Current Technical Systems for Monitoring of Equipment Work in the Logging and Woodworking Production. XXth International Conference for Young Scientists 2011, University of Forestry, Sofia, pp. 63–66.
6. Marinov B., (2005). Shock Wave Vibration Processes Investigation on Certain Classes Circular Machines, 10th Jubilee National Congress on Theoretical and Applied Mechanics, Varna, 13-16 September, Proceeding, pp. 63–69.
7. Obreshkov P., (1997). Woodworking machines. Sofia.
8. Veits V., A. Kochura, A. Martinenko, (1971). Dynamical investigations of drive machines – Moscow.
9. Vlasev, V., G. Vukov. (2002). Errors in Straightening Workpieces on 4-sides Throughfeed Planer Moulders. Woodworking and Furniture Manufacturing № 1, Sofia, pp. 5–7.
10. Vukov, G., J. Gochev, V. Slavov, (2010). Torsional Vibrations in the Saw Unit of a Kind of Circular Saws. Mechanical-Mathematical Model. Proceedings Scientific Papers, Third Scientific and Technical Conference „Innovation in Forest Industry and Engineering Design“, Sofia, pp. 185–188.
11. www.mathematica.com